MATH4105 General Relativity Problem Sheet 8

We're going to try and understand some features of the current cosmological model. Use the accompanying notes where necessary.

I. QUESTIONS

1/ As the universe expands, first radiation, then matter then cosmological constant dominates the energy density. What is the scale factor of the universe when $\rho_\Lambda = \rho_m$? When $\rho_m = \rho_r$?

2/ Use the Saha equation to estimate the temperature at which the concentration of protons is equal to the concentration of hydrogen in the early universe.

Assume that the number of electrons equals the number of protons $n_e = n_p$ and that the baryon energy density is largely made up of protons and Hydrogen atoms that are slowly moving so that $n_p + n_H \simeq \rho_b/m_p c^2$ where m_p is the mass of the proton. (This is not true, about a quarter of all the nuclei in the universe are helium.)

Why is this temperature so much less than 13.6 eV the ionisation energy of hydrogen?

What is the scale factor of the universe when the cosmic microwave background has this temperature? (This time is known as recombination.)

3/ The observed background radiation is not quite isotropic. Suppose that the earth has a velocity v along the positive x-axis. (Choose a scale factor of $a = 1$ for the present time.) Consider a photon having energy equal $E = \bar{\omega} = k_b T$ emitted at recombination so as to arrive at earth today. What is the observed energy as a function of angle?

4/ Suppose the universe were matter dominated prior to recombination What is the spatial radius of the backward light cone of any point in the universe at recombination? Express this analytically in terms of $\dot{a}/a$ at recombination.

5/Given that the universe is flat, what angle does this backward light cone subtend in the sky today? (Again I'm just looking for an analytic expression in terms of the scale factor and Hubble constant at recombination.)

If the universe had positive curvature qualitatively how would this change?

6/ (If you are very keen, not for assessment) Find a numerical value for this angle. This will require numerically integrating the Friedman equation or making some rough approximation.

Clearly the blackbody radiation we see comes from regions that were causally disconnected at recombination. How many fit into the sky roughly?

The blackbody radiation has an excess of fluctuations on roughly this angular scale. Comment on why this could be.