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Sub shot-noise fluctuations in the intensity sum of two coupled beams

G.Yu. Kryuchkyan and K.V. Kheruntsyan
Institute for Physical Research, Armenian Academy of Sciences, Ashtarak-2, 378410, Armenia

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A quantum analysis of intensity fluctuations for the above-threshold parametric four-wave mixing in a cavity, driven by two laser fields, is presented. A new possibility for the reduction of quantum fluctuations below the shot-noise level in the sum of intensities of two cavity-output coupled beams at the driving field frequencies is predicted. The calculation of the corresponding interbeam second-order correlation function displays photon anticorrelation.

1. Introduction

In recent years there have been achieved considerable advances in the investigation of quantum fluctuations in correlated light beams. In the process of nondegenerate parametric oscillation and four-wave mixing (FWM) the photons of signal and idler modes are created in pairs and a positive correlation $\langle \Delta n_1 \Delta n_2 \rangle > 0$ between the photon number fluctuations Δn_1 and Δn_2 of these two modes occurs. As has been shown theoretically [1–4] and demonstrated experimentally [1,5,6], it results in the reduction of quantum fluctuations below the shot-noise level in the intensity difference of these two modes. The coupling between the fluctuations in highly-correlated beams may be used, in particular, for the production of a single light beam with reduced intensity fluctuations by monitoring the intensity fluctuations of the other beam [7]. Another application relates to the absorption spectroscopy below the shot noise limit.

In this paper we present another scheme for reduction of quantum noises, based upon the anticorrelation $\langle \Delta n_1 \Delta n_2 \rangle < 0$ between the photon number fluctuations of two coupled beams. Such situation takes place, for example, in the process of parametric FWM in two laser driving fields. This process is stipulated by the intracavity nonlinear interaction of two pump modes with frequencies ω_1, ω_2 and two signal modes with degenerate frequency ω_0, such that $\omega_1 + \omega_2 = 2\omega_0$. A quantum phenomenological analysis in the below- and above-threshold regimes of oscillation was given in our previous paper [8], where it has been shown, particularly, that this process is quite effective for production of intense one-mode squeezed light with reduced quadrature-phase fluctuations. Nonclassical optical effects in FWM in two driving fields with the inclusion of quantization of nonlinear medium and in the undepletted pump approximation are considered in papers [9,10].

The aim of this paper is to present an analysis of intensity fluctuations in the above-threshold regime of parametric FWM oscillation. As is shown below, in the nonlinear system under consideration the reduction of fluctuations below the shot-noise level occurs in the intensity sum of two pump modes ω_1 and ω_2. A similar prediction concerning quantum correlations in the sum of the intensities of the fundamental and second-harmonic mode in the second-harmonic generation process was done recently in ref. [11].

2. Equations of motion, steady-states and linearization

We consider the following phenomenological model of parametric FWM in two driving fields. A

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nonlinear medium placed inside a suitably tuned ring cavity couples two pump modes of frequencies ω_1, ω_2 and the signal mode of frequency $\omega_0=(\omega_1+\omega_2)/2$ with the wave-vector-matching condition $\boldsymbol{k}_1+\boldsymbol{k}_2=2\boldsymbol{k}_0$. The pump modes are driven by two coherent external driving fields with different frequencies ω_1 and ω_2, equal amplitudes and arbitrary phases. All three cavity modes are damped via cavity losses and treated in quantum way.

The dynamics of modes in the cavity and quantum statistics of this system is described by the stochastic differential c-number Langevin equations for the independent complex field variables α_j, α_j^+, which correspond to the slowly-varying creation and annihilation operators a_j, a_j^+ of three modes ω_j ($j=0$, 1, 2). These equations are obtained in standard manner on the basis of Fokker–Planck equation in positive P-representation and have the following form [8]

$$\begin{aligned}
\dot{\alpha}_0(t)&=-\gamma_0\alpha_0+\kappa\alpha_1\alpha_2\alpha_0^+ +R_0(t)\,,\\
\dot{\alpha}_1(t)&=-\gamma\alpha_1-\tfrac{1}{2}\kappa\alpha_0^2\alpha_2^+ +E\exp(\mathrm{i}\phi_1)+R_1(t)\,,\\
\dot{\alpha}_2(t)&=-\gamma\alpha_2-\tfrac{1}{2}\kappa\alpha_0^2\alpha_1^+ +E\exp(\mathrm{i}\phi_2)+R_2(t)\,,
\end{aligned}\tag{1}$$

where $\frac{1}{2}\kappa$ is the effective coupling constant proportional to the third-order susceptibility, E is the amplitude of the driving fields inside the cavity, ϕ_1 and ϕ_2 are their phases, and γ_0, γ ($\gamma=\gamma_1=\gamma_2$) are the damping constants for the modes ω_0 and ω_1, ω_2, respectively. The gaussian noise terms $R_j(t)$ have the following nonzero correlations

$$\begin{aligned}
\langle R_0(t)\,R_0(t')\rangle&=\kappa\alpha_1\alpha_2\,\delta(t-t')\,,\\
\langle R_1(t)R_2(t')\rangle&=-\tfrac{1}{2}\kappa\alpha_0^2\,\delta(t-t')\,.
\end{aligned}\tag{2}$$

We assume that the cavity has a single input–output port. Then the field operators c_j and b_j around the frequencies ω_j at the cavity input and output are connected with E and intracavity operators α_j as follows (see, e.g., ref. [12])

$$E\exp(\mathrm{i}\phi_k)=\sqrt{2\gamma}\langle c_k\rangle \quad (k=1,2)\,,\tag{3}$$

$$b_j=\sqrt{2\gamma_j}\alpha_j-c_j \quad (j=0,1,2)\,.\tag{4}$$

In the semiclassical limit the stable steady-state solutions are $\alpha_j^0=|\alpha_j^0|\exp(\mathrm{i}\psi_j^0)$, $(\alpha_j^0)^*=(\alpha_j^0)^+$ of set (1) in the above-threshold regime $E>E_t=\gamma_0\sqrt{\gamma/\kappa}$ (E_t is the threshold value of E) are following [8].

(i) In the region $1<\epsilon<2$, where $\epsilon=E/E_t$, the stable steady-state solution for the mode amplitudes is

$$|\alpha_0^0|=\sqrt{(2\gamma/\kappa)(\epsilon-1)}\,,\quad |\alpha_1^0|=|\alpha_2^0|=\sqrt{\gamma_0/\kappa}\,,\tag{5}$$

(ii) In the region $\epsilon>2$ the stable steady-state amplitudes are

$$\begin{aligned}
|\alpha_0^0|&=\sqrt{2\gamma/\kappa}\,,\\
|\alpha_1^0|&=\tfrac{1}{2}\sqrt{\gamma_0/\kappa}\,(\epsilon+\sqrt{\epsilon^2-4})\,,\\
|\alpha_2^0|&=\tfrac{1}{2}\sqrt{\gamma_0/\kappa}\,(\epsilon-\sqrt{\epsilon^2-4})\,,
\end{aligned}\tag{6}$$

or

$$\begin{aligned}
|\alpha_1^0|&=\tfrac{1}{2}\sqrt{\gamma_0/\kappa}\,(\epsilon-\sqrt{\epsilon^2-4})\,,\\
|\alpha_2^0|&=\tfrac{1}{2}\sqrt{\gamma_0/\kappa}\,(\epsilon+\sqrt{\epsilon^2-4})\,.
\end{aligned}$$

In both cases (i) and (ii) the corresponding steady-state phases of three modes are

$$\psi_1^0=\phi_1\,,\quad \psi_2^0=\phi_2\,,\quad \psi_0^0=(\phi_1+\phi_2)/2\,.\tag{7}$$

The values $\epsilon=1$, 2 are instability points, since the all steady-state solutions are unstable at these points. The below-threshold solution and linearizated treatment of quantum fluctuations as well as the quadrature-phase squeezing spectra for each of three modes above threshold are considered in ref. [8]. In the present paper we shall consider the phase insensitive intensity fluctuations in the above threshold regime.

To analyze the quantum fluctuations above threshold we transform the set of equations (1) to the new variables $n_j=\alpha_j\alpha_j^+$, $\psi_j=(1/2\mathrm{i})\ln(\alpha_j/\alpha_j^+)$ and, introducing small fluctuations $\Delta n_j(t)=n_j(t)-n_j^0$ ($n_j^0=|\alpha_j^0|^2$), $\Delta\psi_j(t)=\psi_j(t)-\psi_j^0$, linearize the equations for $n_j(t)$, and $\psi_j(t)$ about the steady-state values n_j^0 and ψ_j^0. The Fourier-transformed linearized equations of motion for the photon number fluctuations Δn_j in the matrix form are

$$(\mathbf{A}-\mathrm{i}\omega\mathbf{I})\,\Delta n(\omega)=F^0(\omega)\,,\tag{8}$$

$$\mathbf{A}=\begin{bmatrix} 0 & -\dfrac{\gamma_0 n_0^0}{n_1^0} & -\dfrac{\gamma_0 n_0^0}{n_2^0} \\ \gamma_0 & \gamma & \dfrac{\gamma_0 n_0^0}{2n_2^0} \\ \gamma_0 & \dfrac{\gamma_0 n_0^0}{2n_1^0} & \gamma \end{bmatrix}, \tag{9}$$

where $\mathbf{I}$ is the identity matrix, $\Delta n(\omega)=[\Delta n_0(\omega), \Delta n_1(\omega), \Delta n_2(\omega)]^{\mathrm{T}}$, $F^0(\omega)=[F_0^0(\omega), F_1^0(\omega), F_2^0(\omega)]^{\mathrm{T}}$. The nonzero correlations of noise terms are

$$\langle F_0^0(\omega)\,F_0^0(\omega')\rangle=-2\langle F_1^0(\omega)\,F_2^0(\omega')\rangle$$
$$=2\gamma_0 n_0^0\,\delta(\omega+\omega')\,. \tag{10}$$

The results for the mean photon number per unit time $N_j=\langle b_j^+ b_j\rangle$ for each of three cavity-output fields around the frequency ψ_j are following. For the coherent components of the output field amplitudes we have from eq. (4)

$$b_{1,2}^0=\langle b_{1,2}\rangle=\sqrt{2\gamma}\,\alpha_{1,2}^0-\langle c_{1,2}\rangle\,,\quad b_0^0=\sqrt{2\gamma_0}\,\alpha_0^0\,. \tag{11}$$

In the linear approximation $N_j=|b_j^0|^2$, and we obtain in the region $1<\epsilon<2$:

$$N_0=\frac{4\gamma\gamma_0}{\kappa}(\epsilon-1)\,,\quad N_1=N_2=\frac{2\gamma\gamma_0}{\kappa}(1-\epsilon/2)^2\,; \tag{12a}$$

and in the region $\epsilon>2$:

$$N_0=\frac{4\gamma\gamma_0}{\kappa}\,,\quad N_1=N_2=\frac{2\gamma\gamma_0}{\kappa}(\epsilon^2/4-1)\,. \tag{12b}$$

It should be pointed out that because of interference of the amplitudes $\alpha_{1,2}^0$ and $\langle c_{1,2}\rangle$ in the quantities N_1 and N_2 the latters coincide in the whole region $\epsilon>1$, though the mean photon numbers for the pump modes inside the cavity $|\alpha_2^0|^2$ and $|\alpha_2^0|^2$ differ in the region $\epsilon>2$.

3. Quantum noise reduction in the intensity fluctuation spectra

The following scheme of experimental measurements is considered. Two photodetectors measure the intensities of the cavity-output fields around the pump mode frequencies ω_1, ω_2, and the fluctuation spectrum

$$P_\pm(\omega)=2\int_0^{+\infty}\mathrm{d}\tau\cos\omega\tau\,\langle i_\pm(0), i_\pm(\tau)\rangle \tag{13}$$

of the sum or difference of corresponding photocurrents $i_\pm=i_1\pm i_2$ is being analyzed. The photocurrent correlator $\langle i_\pm(0),\ i_\pm(\tau)\rangle=\langle i_\pm(0)\ \ i_\pm(\tau)\rangle-\langle i_\pm(0)\rangle\,\langle i_\pm(\tau)\rangle$, in accordance with a standard theory of photodetection [13,14], is

$$\langle i_\pm(0), i_\pm(\tau)\rangle=\langle i(0)\,i(\tau)\rangle_{\mathrm{sh}}$$
$$+(Q\eta)^2\langle{:}N_\pm(0), N_\pm(\tau){:}\rangle\,, \tag{14}$$

where $\langle i(0)\,i(\tau)\rangle_{\mathrm{sh}}$ is the shot-noise term, which is the same for the photocurrent sum and difference and proportional to the sum of two field intensities, Q is the total charge per photopulse, η is the dimensionless efficiency of detectors $(0<\eta\leqslant 1)$, :: denotes normal ordering, and the operators N_+, N_- are

$$N_\pm=b_1^+b_1\pm b_2^+b_2\,.$$

At the calculation of correlator $\langle N_\pm(0), N_\pm(\tau)\rangle$ in the lowest order in small fluctuations the contribution of phase fluctuations is cancellated and the resulting expression reduces to

$$\langle N_\pm(0), N_\pm(\tau)\rangle=2\gamma(b^0)^2$$
$$\times\left[\frac{\langle\Delta n_1(0)\,\Delta n_1(\tau)\rangle}{n_1^0}+\frac{\langle\Delta n_2(0)\,\Delta n_2(\tau)\rangle}{n_2^0}\right.$$
$$\left.\pm\left(\frac{\langle\Delta n_1(0)\,\Delta n_2(\tau)\rangle}{\sqrt{n_1^0 n_2^0}}+\frac{\langle\Delta n_2(0)\,\Delta n_1(\tau)\rangle}{\sqrt{n_1^0 n_2^0}}\right)\right], \tag{15}$$

where $b^0=|b_1^0|=|b_2^0|$.

Converting eq. (13) into frequency space, and using eqs. (14), (15) and δ-function properties of noise correlators, one can obtain

$$P_\pm(\omega)=P_{\mathrm{sh}}\left[1+\eta\gamma\sum_{k=1,2}\frac{\langle\Delta n_k(-\omega)\,\Delta n_k(\omega)\rangle}{n_k^0}\right.$$
$$\pm\eta\gamma\left(\frac{\langle\Delta n_1(-\omega)\,\Delta n_2(\omega)\rangle}{\sqrt{n_1^0 n_2^0}}\right.$$
$$\left.\left.+\frac{\langle\Delta n_2(-\omega)\Delta n_1(\omega)\rangle}{\sqrt{n_1^0 n_2^0}}\right)\right], \tag{16}$$

where we have used the frequency-independent simplified expression for the shot-noise term [14]

$$P_{sh} = \eta Q^2 (N_1 + N_2)\ .$$

The calculation of the second-order averages in eq. (16) on the basis of solution of eq. (8) results in the following final expression for the normalized fluctuation spectra

$$\frac{P_\pm(\omega)}{P_{sh}} = 1 + \frac{4\eta}{d(\omega)} \Big\{ r^2 [p(1+q) + 4q(p-1)] + [pr^2 + 2q(1-2r)](\omega/\gamma)^2 \Big\} \pm \frac{4\eta\sqrt{q}}{d(\omega)} \Big\{ 2r^2(1-p-3q) + [2r^2 + 2r(p-q-1)](\omega/\gamma)^2 - (\omega/\gamma)^4 \Big\}\ , \qquad (17)$$

where the quantity $d(\omega) = |\det(\mathbf{A} - i\omega\mathbf{I})|^2/\gamma^6$ is equal to

$$d(\omega) = 4[r(p-2q) - (\omega/\gamma)^2]^2 + (\omega/\gamma)^2 [1 + 2rp - q - (\omega/\gamma)^2]^2\ ,$$

and the following notations are used

$$r = \frac{\gamma_0}{\gamma}\ , \quad p = \frac{\gamma_0}{2\gamma}\left(\frac{n_0^0}{n_1^0} + \frac{n_0^0}{n_2^0}\right)\ , \quad q = \frac{\gamma_0^2 (n_0^0)^2}{4\gamma^2 n_1^0 n_2^0}\ .$$

The result (17) is written in a general form and it describes the spectrum of fluctuations in the intensity photocurrent sum and difference in both above-threshold regions (i) and (ii). In each of these regions the stable steady-state amplitudes $|\alpha_j^0|$ are different, and the parameters p and q are equal to

$$\text{(i) } 1 < \epsilon < 2: \quad p = 2(\epsilon - 1)\ , \quad q = (\epsilon - 1)^2\ , \qquad (18a)$$

$$\text{(ii) } \epsilon < 2: \quad p = \epsilon^2 - 1\ , \quad q = 1\ . \qquad (18b)$$

The spectrum of fluctuations in the photocurrent sum for the case of ideal photodetectors with $\eta = 1$ is represented in fig. 1 for different values of ϵ and r. The reduction of fluctuations below the shot-noise level ($0 < P_+(\omega)/P_{sh} < 1$) occurs at the sideband frequencies of spectrum and absent at zero frequency. The maximum effect is close to perfect (100%) reduction in the neighborhood of $\epsilon = 2$ and for small values of parameter $r = \gamma_0/\gamma \ll 1$. The dependence of fluctuation spectrum $P_+(\omega_{opt})/P_{sh}$ at the points of minima $\omega = \omega_{opt}$ on parameter ϵ is plotted in fig. 2 for different r. In the region $\epsilon > 2$ the size of fluctuations at ω_{opt} tends to the shot-noise level with increase of ϵ and the reduction is lost.

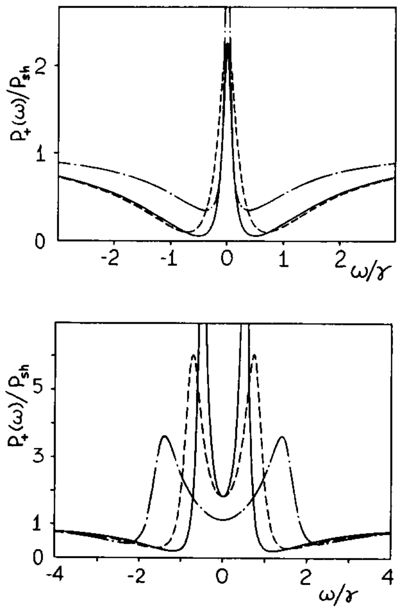

Fig. 1. The photocurrent fluctuation spectrum for the intensity sum $P_+(\omega)/P_{sh}$ versus ω/γ: (a) $\epsilon = 1.3$, $r = 0.05$ (-·-); $\epsilon = 1.8$, $r = 0.05$ (—); $\epsilon = 1.8$, $r = 0.1$ (---); (b) $\epsilon = 3$, $r = 0.1$ (---); $\epsilon = 3$, $r = 0.05$ (—); $\epsilon = 6$, $r = 0.05$ (-·-).

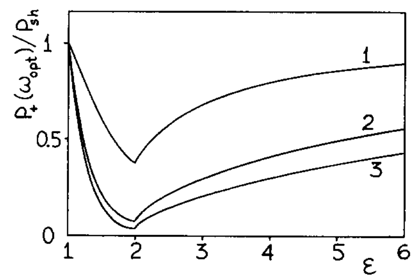

Fig. 2. The dependence of $P_+(\omega_{opt})/P_{sh}$ on parameter ϵ: $r = 0.5$ curve (1), $r = 0.1$ curve (2), $r = 0.05$ curve (3).

The analysis of expression (17) for $P_-(\omega)$ shows that fluctuation suppression for the photocurrent difference is absent.

The reduction of fluctuation in the sum of intensities may be explained in the following way. It is known that the correlation between the instantaneous intensity fluctuations for two coherent driving fields at the cavity input is absent, $\langle \Delta n_1^{\rm in}(t)\, \Delta n_2^{\rm in}(t)\rangle = 0$. As a result of the nonlinear four-photon interaction in the above-threshold regime, when the pump depletion is taken into account, the two beams acquire correlated statistical properties, which are characteristic for two-photon absorption. Namely, the correlation between the intensity fluctuations of two pump fields becomes negative $\langle \Delta n_1(t)\, \Delta n_2(t)\rangle < 0$ (for certain values of parameters ϵ and r). As a result the reduction of quantum fluctuations below the shot-noise level, as it is seen from eqs. (15), (16), is realized for the sum of pump intensities.

The quantity $\langle \Delta n_1(t)\, \Delta n_2(t)\rangle$ is connected in the small fluctuation approximation with the equal-time interbeam second-order correlation function $g_{12}^{(2)}(0)$ as follows

$$g_{12}^{(2)}(0) = \frac{\langle \alpha_1^+(t)\alpha_2^+(t)\alpha_2(t)\alpha_2(t)\rangle}{\langle \alpha_1^+\alpha_1\rangle\langle \alpha_2^+\alpha_2\rangle} = 1 + \frac{\langle \Delta n_1(t)\, \Delta n_2(t)\rangle}{n_1^0 n_2^0}, \tag{19}$$

and it may be calculated by the formula

$$\langle \Delta n_1(t)\, \Delta n_2(t)\rangle = \frac{1}{4\pi}\int_{-\infty}^{+\infty} d\omega \times [\langle \Delta n_1(-\omega)\, \Delta n_2(\omega)\rangle + \langle \Delta n_2(-\omega)\, \Delta n_1(\omega)\rangle]. \tag{20}$$

Using the solutions of eq. (8) and calculating the second-order averages in eq. (20) we obtain at the region $1<\epsilon<2$:

$$\langle \Delta n_1(t)\, \Delta n_2(t)\rangle = -\frac{\gamma_0}{\kappa}\frac{\epsilon-(1+r)(2-\epsilon)}{2\epsilon(2-\epsilon)}, \tag{21}$$

and at the region $\epsilon>2$ for the case $\epsilon^2 \gg 1$:

$$\langle \Delta n_1(t)\, \Delta n_2(t)\rangle = -\frac{\gamma_0}{\kappa}\frac{(1+r-r^2)\epsilon^2 - r}{\epsilon^2(1+r\epsilon^2)}. \tag{22}$$

The expressions (21), (22) allow to determine immediately the values of parameter ϵ and r, for which an interbeam anticorrelation $g_{12}^{(2)}(0) < 1$ is realized. It should be noted, however, that this anticorrelation is insufficient for the reduction of quantum noises below the shot noise level in the intensity sum due to the presence of intrabeam intensity fluctuation terms in eqs. (15) and (16), which are always positive.

References

[1] S. Reynaud, C. Fabre and E. Giacobino, J. Opt. Soc. Am. B 4 (1987) 1520.
[2] R. Horowicz, M. Pinard and S. Reynaud, Optics Comm. 61 (1987) 142.
[3] A.S. Lane, M.D. Reid and D.F. Walls, Phys. Rev. A 38 (1988) 788.
[4] W. Zhang and D.F. Walls, Optics Comm. 79 (1990) 397.
[5] T. Debuisschert, S. Reynaud, A. Heidmann, E. Giacobino and C. Fabre, J. Quantum Optics 1 (1989) 3.
[6] M. Vallet, M. Pinard and G. Grynberg, Europhys. Lett. 11 (1990) 739.
[7] J. Mertz, A. Heidmann, C. Fabre and S. Reynaud, Phys. Rev. Lett. 64 (1990) 2897.
[8] G.Yu. Kryuchkyan and K.V. Kheruntsyan, J. Quantum Optics (1992) in press.
[9] G.Yu. Kryuchkyan, Zh. Exp. Teor. Fiz. 99 (1991) 1416; J. Quantum Optics 3 (1991) 209.
[10] G.Yu. Kryuchkyan and K.V. Kheruntsyan, Optics Comm. 87 (1992) 181.
[11] R.J. Horowics, Europhys. Lett. 10 (1989) 537.
[12] M.J. Collet and C.W. Gardiner, Phys. Rev. A 30 (1984) 1386.
[13] R.J. Glauber, in: Quantum Optics and Electronics, eds. C. de Witt, A. Blandim and C. Cohen-Tannouji (Gordon & Breach, New York, 1965).
[14] H.J. Carmichael, J. Opt. Soc. Am. B 4 (1987) 1588.