Optics Communications 87 (1992) 181–185
North-Holland

OPTICS
COMMUNICATIONS

Squeezing spectrum for radiation of atoms in two laser fields

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Received 9 April 1991; revised manuscript received 6 August 1991

Nonclassical properties of the radiation from an ensemble of two-level atoms interacting with a bichromatic pump field in an optical cavity are studied. In particular, the intensity, squeezing spectrum and the second-order correlation function for the cavity output field are calculated.

1. Introduction

Over the past few years, properties of nonclassical states of light as well as different methods for their production have been widely discussed. In most works, both theoretical and experimental ones, monochromatic continuous-wave pump fields are considered to obtain the nonclassical light, including squeezed states [1]. Several papers are also devoted to problems of squeezed light pulses [2].

In this paper a new possibility for single mode squeezed light generation by coupling an ensemble of two-level atoms with a bichromatic laser field in an optical cavity is considered. This intense field is treated classically and chosen in the following form

$$E(t)=E_0\ \mathrm{Re}\{\exp[-\mathrm{i}(\omega_0+\delta)t-2\mathrm{i}\phi] +\exp[-\mathrm{i}(\omega_0-\delta)t]\}\ . \tag{1}$$

It contains two components with equal amplitudes $E_0/2$, relative phase 2ϕ and frequencies $\omega_0+\delta$, $\omega_0-\delta$, symmetrically detuned from the atomic resonance frequency ω_0.

The dynamics of a bichromatically driven atom has been considered in a series of works [3–8]. The spectrum of resonance fluorescence of an atom in the bichromatic field (1) has a fine structure with peaks at the frequencies $\omega_q=\omega_0+q\delta$, $q=0$, ± 1, ± 2, ..., provided that the detuning δ is much greater than the spontaneous width γ of the excited atomic level [4,5].

As it is shown below, the mode with frequency equal to the resonance fluorescence central line ω_0 is excited into a squeezed state in the presence of an optical cavity. The excitation of the ω_0 mode by the atoms in the bichromatic field occurs due to the process of two-photon spontaneous radiation. There is a strong pair correlation between the photons of the ω_0-mode [5], and this correlation is manifested in the fluctuations of the quadrature field amplitude of the ω_0-mode.

Note, that the situation in the present problem differs from the well-known scheme of squeezed light generation in the process of nondegenerate four-wave mixing [9], where the two weak-intensity sidebands are generated by the central high intensity pump field. For the case discussed here, one weak intensity mode ω_0 is excited by the influence of two intense pump fields with frequencies $\omega_0+\delta$ and $\omega_0-\delta$.

The aim of present paper is to calculate the intensity, squeezing spectrum and temporal second-order correlation function of the cavity output light around the frequency ω_0.

2. Matrix elements of atomic transitions in a bichromatic laser field

The theoretical consideration of the problems listed in the introduction is carried out on the basis of quantum Langevin equations for the radiated field operators. The coefficients of these equations are calculated within the quasi-energy state representation of the system "two-level atom + bichromatic laser field". The description in detail of such an approach as applied to four-wave mixing is discussed in papers [10,11].

Generalizing the results of papers [3–5] to the case of arbitrary relative phase ϕ in eq. (1), one can obtain the following two linearly independent quasi-energy wavefunctions of the two-level atom in the bichromatic pump field (1)

$$\Phi_1(t)=\cos[(\xi/2)\sin(\delta t+\phi)]\varphi_1 -\mathrm{i}\sin[(\xi/2)\sin(\delta t+\phi)] \times\exp[-\mathrm{i}\omega_0 t-\mathrm{i}(\phi+\varphi)]\,\varphi_2\,,$$
$$\Phi_2(t)=-\mathrm{i}\sin[(\xi/2)\sin(\delta t+\phi)] \times\exp[\mathrm{i}(\phi+\varphi)]\,\varphi_1 +\cos[(\xi/2)\sin(\delta t+\phi)]\exp(-\mathrm{i}\omega_0 t)\,\varphi_2\,, \qquad (2)$$

where $\xi=2V_0/\delta$, $V_0=|\boldsymbol{d}E_0/\hbar$, $\boldsymbol{d}=\langle\varphi_2|\boldsymbol{d}|\varphi_1\rangle=|\boldsymbol{d}|\exp(\mathrm{i}\varphi)$, φ_1, φ_2 are the wavefunctions of the non-perturbed atom and we assume that the field (1) is linearly polarized. In obtaining (2) the rotating-wave approximation under conditions $\delta\ll\omega_0$, $V_0\ll\omega_0$ has been used. The wavefunctions Φ_1, Φ_2 coincide with the states φ_1, φ_2 as $\xi\to 0$.

For the case $\gamma t\ll 1$ the spectral lines of radiation of the system "atom+bichromatic field" are determined by the negative- and positive-frequency components of the matrix elements of the dipole transitions between the states $|\Phi_1\rangle$, $|\Phi_2\rangle$

$$\boldsymbol{D}_{ij}(t)=\langle\Phi_i|\boldsymbol{d}|\Phi_j\rangle=\boldsymbol{D}_{ij}^{(-)}(t)+\boldsymbol{D}_{ij}^{(+)}(t)\quad (i,j=1,2)\,,$$
$$\boldsymbol{D}_{ij}^{(-)}(t)=\sum_q \boldsymbol{d}_{ij}^{(q)}\exp[-\mathrm{i}(\omega_q t+q\phi)-\mathrm{i}(\phi+\varphi)]\,,$$
$$\boldsymbol{D}_{ij}^{(+)}=(\boldsymbol{D}_{ij}^{(-)})^*\,, \qquad (3)$$
$$\boldsymbol{d}_{11}^{(q)}=-\boldsymbol{d}_{22}^{(q)}=(\boldsymbol{d}^*/4)J_q(\xi)(1-\cos\pi q)\,,$$
$$\boldsymbol{d}_{12(21)}^{(q)}=\frac{\boldsymbol{d}}{2}\left(\delta_{q0}\pm J_q(\xi)\,\frac{1+\cos\pi q}{2}\right) \times\exp[\pm\mathrm{i}(\phi+\varphi)]\,, \qquad (4)$$

where $J_q(\xi)$ are Bessel functions, and $\omega_q=\omega_0+q\delta$, $q=0,\pm 1,\pm 2,\ldots$.

3. Intracavity mode dynamics

The following scheme to obtain single-mode squeezed light is considered below. Within the ring cavity the system of identical atoms is pumped by the bichromatic driving field with the frequencies of two components $\omega_1=\omega_0+\delta$, $\omega_2=\omega_0-\delta$, which has the form (1) inside the cavity. We consider the case of a single cavity-mode excitation with the resonant frequency equal to the resonance fluorescence central line ω_0, such that $\omega_1+\omega_2=2\omega_0$ ($\boldsymbol{k}_1+\boldsymbol{k}_2=2\boldsymbol{k}_0$). The frequencies ω_1 and ω_2 of two pump fields are located away from the cavity resonance. The driving field incident to the cavity is written as follows

$$\boldsymbol{E}_{\mathrm{in}}(t)=\boldsymbol{E}_1\,\mathrm{Re}\exp(-\mathrm{i}\omega_1 t-\mathrm{i}\phi_1) +\boldsymbol{E}_2\,\mathrm{Re}\exp(-\mathrm{i}\omega_2 t-\mathrm{i}\phi_2)\,. \qquad (5)$$

Then the field inside the cavity would have the form (1) with $\boldsymbol{E}_0=\sqrt{T_1}\,\boldsymbol{E}_1=\sqrt{T_2}\,\boldsymbol{E}_2$ and $2\phi=\phi_1-\vartheta_1$ if we choose $\phi_2-\vartheta_2=0$, where T_1, T_2 are the transmission coefficients of the input mirror and ϑ_1, ϑ_2 are the phase changes on transmission for the ω_1 and ω_2 components, respectively.

The weak ω_0-mode spontaneously excited at the pump fields propagation direction in the atomic medium is described by the slowly varying creation and annihilation operators $a^\dagger(t)$, $a(t)$. The Langevin equation of motion for this mode, derived for a good cavity, where the atomic variables can be adiabatically eliminated, has the following form

$$\mathrm{d}a(t)/\mathrm{d}t=-(\alpha+\Gamma)a(t)+\mu a^\dagger(t)+f(t)\,, \qquad (6)$$

(and complex conjugate equation), where α is the nonlinear polarizability of the atomic medium for the ω_0-mode in the presence of the bichromatic pump field, Γ is the cavity damping constant for the ω_0-mode ($\Gamma\ll\delta$), μ is the coupling constant between the conjugate modes, and $f(t)$ is a noise operator with zero mean. Eq. (6) is written in a general form with neglect of pump depletion, and it describes the dynamics of the ω_0-mode in a cavity allowing for the mode absorption by the cavity and by the atomic medium, the excitation of the conjugate mode, and the quantum noise caused by spontaneous emission and vacuum fluctuations of the electromagnetic field.

The standard method for calculating the coefficients α, μ and the noise correlations is based on the density matrix technique and on the fluctuation regression theorem. A variant of this method which uses quasi-energy wavefunctions is described in ref. [11], applied to four-wave mixing. The equation for the density matrix operator of the coupled system "atom + bichromatic pump field + quantized radiation field" is given in refs. [5,12]. Using these results with the expressions (3), (4) we find that the coefficient α is equal to

$$\alpha=16\sigma J_0^2(\xi)/[3+J_0(2\xi)][5-J_0(2\xi)]\,, \qquad (7)$$

where $\sigma=4\pi N\omega_0|\boldsymbol{ed}^*|^2/\hbar\gamma$ is the absorption coefficient of the ω_0-mode in the absence of any pump, N is the atomic number density, and $\boldsymbol{e}$ is the polarization vector of the ω_0-mode. The quantity α is real and describes the absorption of the ω_0-mode by the atoms. For the case $\xi\ll 1$ we have $\alpha(\xi)\approx\sigma$.

The calculations show that the coupling coefficient μ becomes zero. This result is specific for the case of a bichromatic pump field in the form (1), and is related to the cancellation of the multiphoton processes of absorption and emission.

The nonzero correlators of the quantum noise operators turn out to be equal to

$$\begin{aligned}\langle f^\dagger(t)f(t')\rangle&=[1+2(\alpha+\Gamma)/\beta]^{-1}\langle f(t)f^\dagger(t')\rangle\\&=\beta\delta(t-t')\,,\\ \langle f(t)f(t')\rangle&=\lambda\delta(t-t')\,,\\ \langle f^\dagger(t)f^\dagger(t')\rangle&=\lambda^*\delta(t-t')\,, \qquad (8)\end{aligned}$$

where the diffusion coefficient β describes the rate of photon emission at frequency ω_0 in the process of resonance fluorescence in the bichromatic field and is equal to

$$\beta=2\sigma\frac{[1+J_0^2(\xi)][3+J_0(2\xi)]-8J_0^2(\xi)}{[3+J_0(2\xi)][5-J_0(2\xi)]}\,. \qquad (9)$$

The coefficient λ describes the spontaneous process of photon pair radiation at the frequency ω_0 and is equal to

$$\lambda=2\sigma\{[1-J_0^2(\xi)]/[5-J_0(2\xi)]\}\exp(2\mathrm{i}\phi)\,. \qquad (10)$$

Note that the coefficient λ contains only the relative phase 2ϕ between the two components of the bichromatic field, and the phase φ of the atomic dipole transition matrix element $\boldsymbol{d}$ is absent in the final results.

Another specific characteristic of the radiation of the considered bichromatically driven atomic system in a cavity should be pointed out. As the calculations show [13], the coupling coefficients μ_{ij} between the two arbitrary cavity resonant modes ω_i and ω_j are equal to zero if the following condition is satisfied, $\omega_i+\omega_j=2\omega_0+p\delta$, where p is an odd number or $p=0$ (the case with $p=0$ gives $\mu_{00}\equiv\mu=0$). Therefore the results of this paper could be applied, when the cavity is tuned to the frequency ω_0 and to the frequencies of the driving field $\omega_1=\omega_0+\delta$, $\omega_2=\omega_0-\delta$ too (providing the conservation of the form (1) for the driving field inside the cavity by the suitable choice of the cavity mirrors). In this case we have $\mu_{01}=\mu_{02}=0$, and the equation of motion for the ω_0-mode will not contain the operators of the ω_1 and ω_2 modes and it will remain in the same form (6) with $\mu=0$.

4. Intensity of the output field

Using the solutions of eq. (6) (with $\mu=0$) and the correlators (8), in terms of Fourier-transformed frequency components we obtain the following results for the second-order expectation values

$$\langle a^\dagger(\omega)\,a(\omega')\rangle=\{\beta/[(\alpha+\Gamma)^2+\omega^2]\}\,\delta(\omega-\omega')\,, \qquad (11)$$

$$\langle a(\omega)\,a(\omega')\rangle=\{\lambda/[(\alpha+\Gamma)^2+\omega^2]\}\delta(\omega+\omega')\,. \qquad (12)$$

The corresponding temporal means are

$$\langle a^\dagger(t+\tau)\,a(t)\rangle=[\beta/2(\alpha+\Gamma)]\exp[-(\alpha+\Gamma)\tau]\,, \qquad (13)$$

$$\langle a(t+\tau)\,a(t)\rangle=[\lambda/2(\alpha+\Gamma)]\exp[-(\alpha+\Gamma)\tau]\,,\quad (\tau>0)\,. \qquad (14)$$

Let us now calculate the intensity of the output field. Using the well-known relation between the cavity output electric-field operators $E^\pm(t)$ and the intracavity operators $a(t)$, $a^\dagger(t)$ [14] for the case of a single cavity-output mirror, we obtain the following result for the total intensity of the output field centred on the frequency ω_0

$$I=(4\pi\hbar\omega_0/cS)\Gamma\langle a^\dagger a\rangle=(4\pi\hbar\Gamma\omega_0/cS)$$

$$\times\frac{[1+J_0^2(\xi)][3+J_0(2\xi)]-8J_0^2(\xi)}{16J_0^2(\xi)+(\Gamma/\sigma)[5-J_0(2\xi)][3+J_0(2\xi)]}, \tag{15}$$

where S is the transverse area, defined by the cavity and detection optics.

For the corresponding spectral intensity we have

$$I(\omega)=(8\pi\hbar\Gamma\omega_0/cS)$$

$$\times\mathrm{Re}\int_0^{+\infty}\mathrm{d}\tau\exp[-\mathrm{i}(\omega-\omega_0)\tau]\ \langle a^\dagger(t+\tau)\ a(t)\rangle$$

$$=2I(\alpha+\Gamma)/[(\alpha+\Gamma)^2+(\omega-\omega_0)^2]\ . \tag{16}$$

The peak value of the spectral intensity in the center of the Lorentz line ($\omega=\omega_0$) is

$$I(\omega_0)(2\pi\hbar\omega_0/cS)^{-1}$$

$$=(4\Gamma/\sigma)[5-J_0(2\xi)][3+J_0(2\xi)]$$

$$\times\{[1+J_0^2(\xi)][3+J_0(2\xi)]-8J_0^2(\xi)\}$$

$$\times\{16J_0^2(\xi)+(\Gamma/\sigma)[5-J_0(2\xi)][3+J_0(2\xi)]\}^{-2}\ . \tag{17}$$

The dependence of intensities (15), (17) on the pump intensity parameter $\xi=V_0/\delta$ is plotted in figs. 1, 2. The maxima of intensities I, $I(\omega)$ lie near those values of ξ for which the absorption coefficient $\alpha(\xi)=0$ and the atomic medium is transparent.

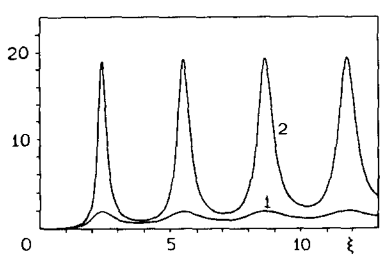

Fig. 1. Intensity $I/(4\pi\hbar\Gamma\omega_0/cS)$ versus parameter $\xi=2V_0/\delta$ for two values of ratio Γ/σ: (1) $\Gamma/\sigma=0.1$ and (2) $\Gamma/\sigma=0.01$.

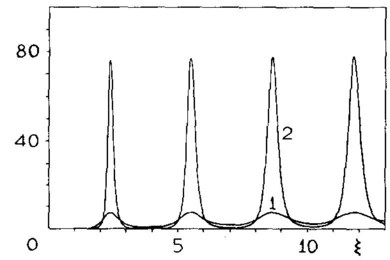

Fig. 2. Peak value $I(\omega_0)/(2\pi\hbar\omega_0/cS)$ of spectral intensity as function of ξ for Γ/σ as in fig. 1.

5. Squeezing spectrum and correlation function

The other quantity of interest is the spectrum of the quadrature-amplitude fluctuations for the output radiation field. This quantity is defined by the expression

$$S(\omega,\vartheta)=4\Gamma\mathrm{Re}\int_0^{+\infty}\mathrm{d}\tau\exp(-\mathrm{i}\omega\tau)$$

$$\times[\langle :X_\vartheta(t+\tau)\ X_\vartheta(t):\rangle$$

$$-\langle X_\vartheta(t+\tau)\rangle\langle X_\vartheta(t)\rangle]\ ,$$

where $X_\vartheta(t)$ is the quadrature-amplitude operator

$$X_\vartheta(t)=a(t)\exp(-\mathrm{i}\vartheta)+a^\dagger(t)\exp(\mathrm{i}\vartheta)\ . \tag{19}$$

The quantity $S(\omega,\ \vartheta)\geqslant-1$ by definition and squeezing is realized if $S(\omega,\vartheta)<0$. Using the results (13), (14) we obtain

$$S(\omega,\vartheta)=\{4\Gamma(\alpha+\Gamma)/[(\alpha+\Gamma)^2+\omega^2]\}$$

$$\times\langle:(\Delta X_\vartheta)^2:\rangle\ . \tag{20}$$

Here $\langle:(\Delta X_\vartheta)^2:\rangle$ is the normally-ordered intracavity variance of the ω_0-mode quadrature-amplitude

$$\langle:(\Delta X_\vartheta)^2:\rangle=[1/(\alpha+\Gamma)][\beta+|\lambda|\cos(2\vartheta-2\phi)]\ , \tag{21}$$

which can be negative for properly chosen phase ϑ. Note, that due to the coupling coefficient $\mu=0$, the squeezing spectrum is determined merely by the contribution of the noise correlators (8).

The optimal value of the quantity $S(\omega,\vartheta)$, which is realized for the phase $\vartheta=\phi+\pi/2$ and for zero frequency $\omega=0$, is

$$S_{\min}(0) = -(1 - J_0(2\xi)) \times \left(\frac{\sigma}{\Gamma} + \frac{[5 - J_0(2\xi)][3 + J_0(2\xi)]}{16 J_0^2(\xi)} \right)^{-1} . \tag{22}$$

The dependence of this quantity on parameter ξ is plotted in fig. 3. The squeezing is absent for very intense ($\xi \gg 1$) and weak ($\xi \ll 1$) pump fields and for values of ξ determined by the equation $J_0(\xi) = 0$. As follows from eq. (7) the atomic medium is transparent for such values of ξ. For the wide range of intermediate values of ξ there is a significant reduction of fluctuations below the vacuum level. The analysis shows that the maximal squeezing may reach 35% for $\Gamma \approx \alpha(\xi)$. As for the intracavity variance (21), this is minimized for $\Gamma \ll \alpha(\xi)$.

The pair correlation between the photons of the ω_0-mode also comes out in another optical phenomenon – in the intensity interference of the radiated field, described by the second-order correlation function

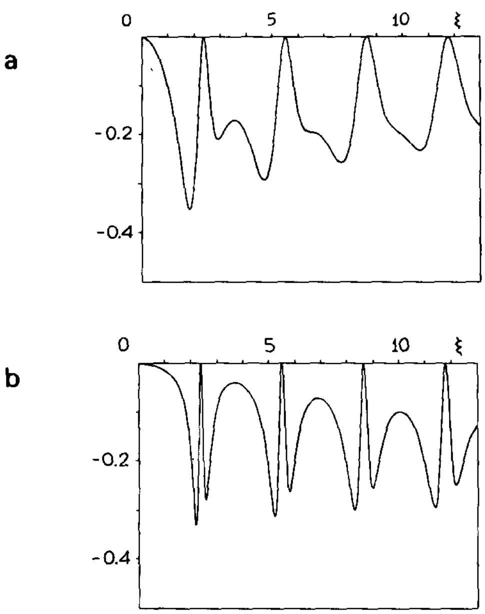

Fig. 3. Peak squeezing $S_{\min}(0)$ versus ξ for: (a) $\Gamma/\sigma = 0.1$ and (b) $\Gamma/\sigma = 0.01$.

$$g^2(\tau) = \langle E^{(-)}(t)\, E^{(-)}(t+\tau) \times E^{(+)}(t+\tau)\, E^{(+)}(t) \rangle / I^2 . \tag{23}$$

Expressing the operators $E^{(\pm)}(t)$ in terms of $a(t)$, $a^\dagger(t)$ and using the gaussian-type noise properties of the vacuum fluctuations to factorize the fourth-order expectation value we obtain (with the help of eqs. (13) and (14))

$$g^{(2)}(\tau) = 1 + \left[1 + \left(\frac{1 - J_0^2(\xi)}{1 - J_0^2(\xi)[5 - J_0(\xi)]/[3 + J_0(2\xi)]} \right)^2 \right] \times \exp\{-2[\Gamma + \alpha(\xi)]\tau\} . \tag{24}$$

For zero delay time $\tau = 0$ the correlation function $g^{(2)}(0) \geqslant 3$ for all values of ξ. For small ξ and $\sigma\tau \ll 1$ the correlation shows photon superbunching and super-poissonian statistics ($g^{(2)} \gg 2$).

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