Quantum correlated twin atomic beams via photodissociation of a molecular Bose-Einstein condensate

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We study the process of photodissociation of a molecular Bose-Einstein condensate as a potential source of strongly correlated twin atomic beams. We show that the two beams can possess nearly perfect quantum squeezing in their relative numbers.

The successful production of Bose-Einstein condensates (BEC) has led to measurements indicating that BECs have coherence properties similar to lasers. This suggests that the next stage in obtaining precision measurements in atom optics is the production of atom beams with sub-Poissonian atom statistics—as has been widely demonstrated in photonics applications. Indeed, there is already some indirect evidence of atom-number squeezing, but experiments and theoretical proposals to date have focused on trapped condensates and phase-sensitive measurements [1–3]. A possible route towards more robust phase-insensitive applications of atomic squeezing is to produce quantum correlated atom laser beams in which the correlations are directly built in the statistics of the particle numbers, as in parametric down-conversion in quantum optics [4]. Quantum correlated or entangled photon pairs from parametric down-conversion are one of the most powerful resources of quantum optics, and our motivation here originates from the intriguing prospect of possessing a *matter-wave* analog of this resource. As well as the immediate possibility of improved atomic interferometry, an exciting prospect would be the development of new tests of quantum measurement theory for massive particles with spacelike separations, since all previous tests using down-conversion methods were restricted to massless photons.

In this paper we propose a robust scheme for achieving strong quantum correlations between two counterpropagating atomic beams, relying on the process of photodissociation of a molecular Bose-Einstein condensate. Experiments towards production of molecular condensates [5,6], together with a number of theoretical studies of coupled atomic-molecular BEC systems are the subject of much intense activity at present [7–9]. We anticipate that the formation of a molecular BEC is a matter of time, and consider it as the starting point for a twin atomic beam experiment with relative atom-number fluctuations reduced below the level predicted by either thermal or coherent (Poissonian) statistics. The method automatically yields two counterpropagating beams through momentum conservation, and is robust against changes in the mode structure, coupling constants, or even absorptive losses, provided they are small.

An important feature of our proposal is that it does not rely on atomic interferometry or local oscillators to generate the resulting correlations, which makes it more practical than recent related proposals [2,3]. In addition, the present scheme is not susceptible to phase noise from self-phase

modulation. In the following analysis, we first analyze a simplified theory of a uniform, nondepleted molecular condensate, then include the effects of molecular trapping, depletion, phase diffusion and atomic losses.

The quantum-field-theory effective Hamiltonian for the atomic ($\hat{\Psi}_1$) and molecular ($\hat{\Psi}_2$) fields, taken for simplicity to be confined to one space dimension, is

$$\hat{H} = \hat{H}_{kin} + \int dx \left\{ \sum_i V_i(x) \hat{\Psi}_i^\dagger \hat{\Psi}_i + \sum_{i \geqslant j} U_{ij} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i \right.$$
$$\left. - i \frac{\chi(t)}{2} [e^{i\omega t} \hat{\Psi}_2^\dagger \hat{\Psi}_1^2 - e^{-i\omega t} \hat{\Psi}_2 \hat{\Psi}_1^{\dagger 2}] \right\}, \quad (1)$$

with the commutation relation $[\hat{\Psi}_i(x,t), \hat{\Psi}_j^\dagger(x',t)] = \delta_{ij}\delta(x - x')$. Here $\hat{H}_{kin}$ stands for the usual kinetic energy term, $V_i(x)$ is the trap potential (including internal energies), $U_{11} \simeq 4\pi\hbar a_1/(Am_1)$ is the atom-atom coupling constant in one dimension, where m_1 is the mass, a is the three-dimensional S-wave scattering length, and A is the confinement area in the transverse direction, with similar results for the molecule-molecule and molecule-atom S-wave interactions. The term proportional to $\chi(t)$ describes a coherent process of molecule-atom conversion via either one- or two-photon (Raman) photodissociation, where $\chi(t)$ is the coupling constant related to the transition matrix element(s) and the amplitude(s) of the photodissociation laser(s) which have an overall frequency (difference) ω.

The time dependence of $\chi(t)$ controls the duration of dissociation process and will be set to $\chi(t) = \chi_0 \theta(t) \theta(t_1 - t)$, where θ is a step function. This means that the dissociation can be stopped after a short-time interval t_1 followed by free evolution of the atomic field. Starting from a pure molecular condensate, the molecular field can be initially described semiclassically, via its initial coherent amplitude $\Psi_2(x,0) = \langle \hat{\Psi}_2(x,0) \rangle$. We assume $n_2(x) = |\Psi_2(x,0)|^2$ is the initial molecular BEC density in a harmonic trap in the Thomas-Fermi limit, with the axial half-length x_0.

The atoms are assumed to be untrapped longitudinally (they may be in an $m = 0$ magnetic sublevel) yet confined transversely (they may be in a transverse optical trap), so that the atomic field can effectively be treated as a free one-dimensional field, initially in a vacuum state. One-dimensional trapping of condensates has been achieved experimentally [10], so this is not unrealistic. In what follows,

we will neglect the atom-atom collisions for simplicity, owing to the fact that we restrict ourselves to short interaction times during which the atomic density does not grow to high values, so that the self-interaction term proportional to U_{11} can be neglected. In addition, we choose the absolute value of the detuning $|\Delta|$ to be always nonzero and much larger than the magnitude of $U_{12}\langle \hat{\Psi}_2^\dagger \hat{\Psi}_2 \rangle$ so that the atom-molecule S-wave scattering is negligible too. The detuning $\Delta = V_1(0) - [V_2(x_0) + \omega]/2$, where $V_2(x_0) = V_2(0) + U_{22}n_2(0)$, is proportional to the energy mismatch between the atomic and molecular fields.

The system has a direct analogy with traveling-wave parametric down-conversion in nonlinear optics [11]. Here the role of χ_0 is played by a nonlinear crystal with second-order susceptibility $\chi^{(2)}$, and a finite interaction time is analogous to the crystal length $L = v\tau_1$ in the direction of propagation, where v is the group velocity of the fundamental (higher frequency) beam. The detuning Δ is analogous to the optical phase mismatch, while the atomic kinetic energy is analogous to dispersion.

To proceed with the analysis we introduce a characteristic length scale d_0 and time scale $t_0 = 2m_1 d_0^2/\hbar$. Next, transform to dimensionless fields, in rotating frames:

$$\hat{\psi}_1(\xi,\tau) = \sqrt{d_0}\,\hat{\Psi}_1(x,t)e^{i[V_2(x_0)+\omega]t/2},$$

$$\hat{\psi}_2(\xi,\tau) = \sqrt{d_0}\,\hat{\Psi}_2(x,t)e^{iV_2(x_0)t},$$

where $\xi = x/d_0$ and $\tau = t/t_0$ are the dimensionless coordinate and time. We also introduce dimensionless detuning $\delta = \Delta t_0$ and coupling $\kappa(t) = \chi(t)t_0/\sqrt{d_0}$.

The Heisenberg equations of motion for the field operators, in dimensionless form are

$$\frac{\partial \hat{\psi}_1(\xi,\tau)}{\partial \tau} = i\frac{\partial^2 \hat{\psi}_1}{\partial \xi^2} - i\delta\hat{\psi}_1 + \kappa\hat{\psi}_2\hat{\psi}_1^\dagger,$$

$$\frac{\partial \hat{\psi}_2(\xi,\tau)}{\partial \tau} = \frac{i}{2}\frac{\partial^2 \hat{\psi}}{\partial \xi^2} - i\hat{v}_2(\xi)\hat{\psi}_2 - \frac{1}{2}\kappa\hat{\psi}_1^2, \qquad (2)$$

together with the Hermitian conjugate equations. We have introduced an effective potential $\hat{v}_2(\xi) = [V_2(\xi d_0) - V_2(\xi_0 d_0)]t_0 + u\hat{\psi}_2^\dagger\hat{\psi}_2$, where $u = U_{22}t_0/d_0$, for notational simplicity.

To gain some insight into the underlying physics of correlated atomic beams, we first consider an idealized and analytically solvable model corresponding to an undepleted and uniform molecular condensate at density $n_2(0)$ that fills the entire space from $-l/2$ to $l/2$, with periodic boundary conditions at $-l/2$ and $l/2$. The atom-molecule coupling $\chi = \chi_0$ is assumed to be constant during the whole evolution time from 0 to τ. In this case the coherent amplitude of the molecular field can be absorbed into an effective gain constant $g = \kappa_0\sqrt{n_2(0)d_0}$ (where $\kappa_0 = \chi_0 t_0/\sqrt{d_0}$), which we assume without loss of generality to be real and positive.

Solutions to the resulting linear set of equations of motion for the atomic field are easily found in momentum space,

where we expand $\hat{\psi}_1(\xi,\tau)$ in terms of single-mode annihilation operators: $\hat{\psi}_1(\xi,\tau) = \Sigma_q \hat{a}_q(\tau)e^{iq\xi}/\sqrt{l}$, where $q = d_0 k$ is a dimensionless momentum. The single-mode operators $\hat{a}_q$ satisfy the usual commutation relations $[\hat{a}_q(\tau),\hat{a}_{q'}^\dagger(\tau)] = \delta_{qq'}$. The corresponding Heisenberg equations of motion have the following solution:

$$\hat{a}_q(\tau) = A_q(\tau)\hat{a}_q(0) + B_q(\tau)\hat{a}_{-q}^\dagger(0),$$

$$\hat{a}_{-q}^\dagger(\tau) = B_q(\tau)\hat{a}_q(0) + A_q^*(\tau)\hat{a}_{-q}^\dagger(0), \qquad (3)$$

where

$$A_q(\tau) = \cosh(g_q\tau) - i\lambda_q\sinh(g_q\tau)/g_q,$$

$$B_q(\tau) = g\,\sinh(g_q\tau)/g_q, \qquad (4)$$

with $\lambda_q \equiv q^2 + \delta$, and $g_q = (g^2 - \lambda_q^2)^{1/2}$. Solutions of this type to the classical counterpart of the operator equations are well known in optics [11], while in quantum optics the operator equations in the context of squeezing of nonlinear propagating fields were studied in Ref. [12].

Knowledge of the initial state of the atomic field, which is the vacuum state with $\hat{a}_q(0)|0\rangle = 0$, allows us to calculate any operator moments at time τ. The parameter g_q is the gain coefficient; if real, it causes a growing correlated output for the momentum component q, while if imaginary it leads to oscillations.

For example, the particle number distribution in momentum space is given by $\langle \hat{a}_q^\dagger(\tau)\hat{a}_q(\tau)\rangle = B_q^2(\tau)$. For $\delta < 0$, the function B_q^2 has two distinct global maxima located at q values where $\lambda_q = 0$. This gives the two most probable momentum values $q_0 = \pm\sqrt{|\delta|}$, corresponding to a zero *effective* phase mismatch term $(q^2 + \delta)$, provided $\delta < 0$. The total average number of atoms $\langle \hat{N}(\tau)\rangle = \Sigma_q\langle \hat{a}_q^\dagger(\tau)\hat{a}_q(\tau)\rangle$ is given by

$$\langle \hat{N}(\tau)\rangle = \sum_q (g/g_q)^2\sinh^2(g_q\tau), \qquad (5)$$

which grows exponentially with τ.

To analyze correlations and relative number squeezing, we define particle number operators $\hat{N}_-(\tau)$ and $\hat{N}_+(\tau)$ containing only negative or positive momentum components, respectively, $\hat{N}_{-(+)}(\tau) = \Sigma_{q<0(q>0)}\hat{a}_q^\dagger(\tau)\hat{a}_q(\tau)$. We next consider the normalized variance $V(\tau)$ of the particle number difference $[\hat{N}_-(\tau) - \hat{N}_+(\tau)]$, which—in the normally ordered form—is given by

$$V(\tau) = 1 + \langle :[\Delta(\hat{N}_- - \hat{N}_+)]^2: \rangle/(\langle \hat{N}_-\rangle + \langle \hat{N}_+\rangle), \qquad (6)$$

where $\Delta\hat{X} \equiv \hat{X} - \langle \hat{X}\rangle$, and $V(\tau) < 1$ implies squeezing of fluctuations below the coherent level—which is due to quantum correlations between $\hat{N}_-$ and $\hat{N}_+$.

Calculating $\langle :(\hat{N}_\pm)^2: \rangle$ and $\langle \hat{N}_-\hat{N}_+\rangle$, and using the fact that $B_q^2 - |A_q|^2 = -1$, gives

$$V(\tau) = 1 + \sum_{q>0} B_q^2 (B_q^2 - |A_q|^2) / \sum_{q>0} B_q^2 = 0, \qquad (7)$$

implying perfect (100%) squeezing in the particle number difference, at least in this idealized calculation.

We now turn to the analysis of a more realistic nonuniform case and include the molecular field depletion, as described by the original coupled operator equations for the fields, Eqs. (2). These are solved numerically using equivalent stochastic (c-number) differential equations in the positive-P representation [13], where we additionally include a coupling to an atomic loss reservoir to describe possible linear losses at a rate γ,

$$\frac{\partial \psi_1}{\partial \tau} = i \frac{\partial^2 \psi_1}{\partial \xi^2} - (\gamma + i\delta)\psi_1 + \kappa \psi_2 \psi_1^+ + \sqrt{\kappa \psi_2}\, \eta_1,$$

$$\frac{\partial \psi_1^+}{\partial \tau} = -i \frac{\partial^2 \psi_1^+}{\partial \xi^2} - (\gamma - i\delta)\psi_1^+ + \kappa \psi_2^+ \psi + \sqrt{\kappa \psi_2^+}\, \eta_1^+,$$

$$\frac{\partial \psi_2}{\partial \tau} = \frac{i}{2} \frac{\partial^2 \psi_2}{\partial \xi^2} - iv(\xi,\tau)\psi_2 - \frac{\kappa}{2}\psi_1^2 + \sqrt{-iu}\,\psi_2 \eta_2,$$

$$\frac{\partial \psi_2^+}{\partial \tau} = -\frac{i}{2} \frac{\partial^2 \psi_2^+}{\partial \xi^2} + iv(\xi,\tau)\psi_2^+ - \frac{\kappa}{2}\psi_1^{+2} + \sqrt{iu}\,\psi_2^+ \eta_2^+. \qquad (8)$$

Here ψ_i and ψ_i^+ are complex stochastic fields corresponding, respectively, to the operators $\hat{\psi}_i$ and $\hat{\psi}_i^\dagger$, $v(\xi,\tau) = [V_2(\xi d_0) - V_2(\xi_0 d_0)]t_0 + u\psi_2 \psi_2^+$ represents the effective potential, and η_i η_i^+ are four real independent delta-correlated Gaussian noise terms: $\langle \eta_i(\xi,\tau)\eta_j(\xi',\tau') \rangle = \langle \eta_i^+(\xi,\tau)\eta_j^+(\xi',\tau') \rangle = \delta_{ij}\delta(\xi - \xi')\delta(\tau - \tau')$.

We consider molecules as an initial coherent field corresponding to Thomas-Fermi inverted parabola for the molecular density. In this case, assuming that $\psi_2(\xi,0)$ is real, we have $\psi_2(\xi,0) = \sqrt{d_0 n_2(\xi)}$, where $n_2(x) = n_2(0)[1 - (\xi/\xi_0)^2]\theta(\xi_0 - |\xi|)$. The molecular condensate axial half-length is denoted via ξ_0, which is determined by the trap geometry, and we assume repulsive molecule-molecule interactions. The time duration for the molecule-atom conversion is controlled via $\kappa(\tau) = \kappa_0 \theta(\tau_1 - \tau)$, so that $\kappa(\tau) = 0$ for $\tau > \tau_1$. Once the dissociation is stopped, we continue the evolution of the resulting atomic field in free space to allow spatial separation of the modes with positive and negative q values.

For spatially separated components, we can introduce a pair of particle number operators,

$$\hat{N}_{-(+)}(\tau) = \int_{-l/2(0)}^{0(l/2)} \hat{\psi}_1^\dagger(\xi,\tau)\hat{\psi}_1(\xi,\tau)d\xi. \qquad (9)$$

Next, we define the normalized variance $V(\tau)$ of the particle number difference $[\hat{N}_-(\tau) - \hat{N}_+(\tau)]$ as before [see Eq. (6)], and evaluate the average values numerically using the
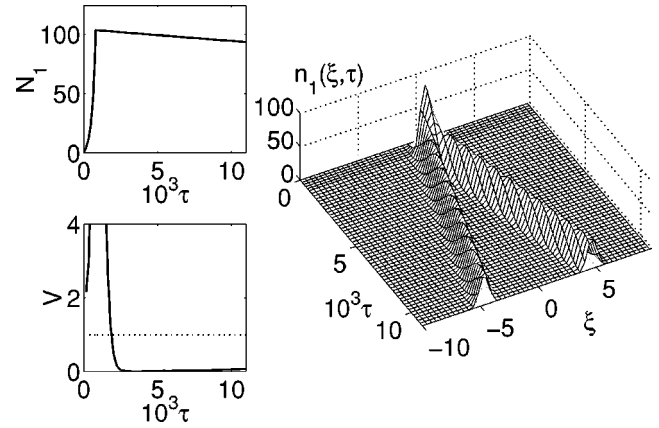

FIG. 1. Atomic density $n_1(\xi,\tau)$, total number of atoms $N_1(\tau)$, and the integrated variance $V(\tau)$, for 40 000 trajectory averages and for: $\delta = -5.017 \times 10^4$, $u = 1.8$, $\kappa_0 = 84$, and $\gamma = 5$. The dimensionless dissociation time in this simulation is $\tau_1 = 8 \times 10^{-4}$. With the choice of the length scale $d_0 = 30\,\mu$m and the mass of a ^{87}Rb atom, we obtain the time scale $t_0 = 2.446$ s, so that $\tau = 0.011$ corresponds to the total time window $t \simeq 27$ ms, while the dissociation time τ_1 scales to $t_1 \simeq 2$ ms.

positive-P Eqs. (8) and the standard correspondence between the normally ordered operator averages and the c-number stochastic averages [13].

The results for $V(\tau)$, the total number of atoms in the two beams $N_1(\tau) = \langle \hat{N}_-(\tau) \rangle + \langle \hat{N}_+(\tau) \rangle$, and the density distribution $n_1(\xi,\tau) = \langle \hat{\psi}_1^\dagger(\xi,\tau)\hat{\psi}_1(\xi,\tau) \rangle$ are represented in Fig. 1, for parameters that are reasonable in present experiments and including a linear atomic loss term as discussed below.

The initial growth of the fluctuations in the atom number difference, during the time interval when the atom-molecule coupling is switched on, is due to the fact that the quantities $\langle N_-(\tau) \rangle$ and $\langle N_+(\tau) \rangle$ each include atoms traveling in opposite directions inside the molecular BEC region. The density distribution in coordinate space at this stage is single peaked, due to the fact that it contains amplified contributions from both momentum components. The fundamental correlation of opposite momentum components is therefore not visible in the atomic density initially. Once, however, the interaction is switched off and the correlated atom pairs fly apart without further parametric amplification, there is a double-peaked distribution, and we see a rapid reduction of the variance below the coherent level, $V(\tau) < 1$.

The physical reason for the correlation is momentum conservation, which requires that each emitted atom with $q > 0$ be accompanied by a partner atom having $q < 0$. In order to conserve energy, this pairing only occurs for $\delta < 0$, which allows the potential energy in the molecule to be converted to atomic kinetic energy for selected modes with q values around $q_0 = \pm\sqrt{|\delta|}$.

As the scheme relies on conservation laws for its operation, it should be insensitive to the exact mode structure. Clearly, quasi-one-dimensional (1D) traps are preferable for reasons of directionality, but we expect similar results even if there is no transverse trap, provided that the molecules are confined in a high aspect-ratio (cigar-shaped) trap to allow gain guiding of the atoms.

Investigations of losses show that these have a minimal effect on squeezing, provided the atoms that are lost are only a fraction of the atoms produced. For example, including an atomic loss term at a rate $\gamma/2 = 10$, corresponding to the loss of $\sim 10\%$ of atoms during the free evolution time interval, gives $V \simeq 0.07$ at $\tau = 0.011$ as shown in Fig. 1. This corresponds to a rather high ($\sim 93\%$) degree of squeezing below the coherent level. Additional effects may occur when there is an atomic potential, or when there is a strong atom-molecule scattering, but this has very little effect on squeezing when $|\delta| \gg |u_a|$, if u_a is the scaled effective atomic potential.

The parameter values of Fig. 1 are derived using a length scale of $d_0 = 30$ μm taken to be equal to the molecular condensate axial half-length R_x, so that $R_x \equiv x_0 = d_0$ and $\xi_0 = 1$. We assume that the molecular BEC is formed in a highly elongated trap and at densities that satisfy the conditions for the crossover from 3D to 1D (see, e.g., Ref. [10]). Assuming that the molecule-molecule scattering length a_2 is of the same order of magnitude as the scattering length of ^{87}Rb atoms, we take $a_2 = a_1 = 5.4$ nm. We also take the molecular BEC linear density at the trap center $n_2(0) = 3.7 \times 10^7$ m^{-1} and an aspect ratio of 100. This implies that the condensate transverse radius is $R_\perp = 0.3$ μm, so that the 3D peak density is about 3.27×10^{19} m^{-3}. The initial total number of molecules is $N_2 = 1.48 \times 10^3$. Using the value of $R_\perp$ to scale out the transverse confinement ($A \to \pi R_\perp^2$), we can next estimate the one-dimensional values of U_{ij} and χ_0. The 1D value of χ_0 is obtained according to $\chi_0 = \chi^{(3D)}/\sqrt{\pi R_\perp^2}$, where we take $\chi^{(3D)} = 2 \times 10^{-7}$ m$^{1/2}$/s [7]. The molecular condensate half-length of 30 μm corresponds to the trap axial oscillation frequency $\omega_x/2\pi = 4.1$ Hz, where $\omega_x = \sqrt{2\hbar U_{22} n_2(0)/(m_2 x_0^2)}$. We choose $\Delta = -2.051 \times 10^4$ s^{-1}, thus assuring that the the atom-molecule S-wave scattering is negligible in this calculation since $|\Delta| \gg |U_{12}|\langle \hat{\Psi}_2^\dagger \hat{\Psi}_2 \rangle$, where U_{12} is estimated using $a_{12} \simeq -9.25$ nm [5]. The final values of the parameters δ, u, and κ_0 that we arrive at are specified in the figure caption, together with the relevant time scales.

In summary, we have shown that photodissociation of a molecular BEC can provide a simple yet robust scheme for quantum squeezing of relative number fluctuations in two counterpropagating atomic beams. The effects of molecular condensate trapping and depletion, molecular self-phase modulation, and atomic absorption have all been included in our calculations. Our method does require high efficiency atom counting techniques [14], which are currently the subject of intensive activity.

Applications may emerge from the use of these quantum-entangled twin beams to produce a single beam with a well-defined particle number, which can be achieved by a destructive measurement on the partner beam. While this should be readily observable, even more subtle experiments may be feasible in future, including possible demonstrations of Einstein-Podolsky-Rosen correlations or Bell inequalities [15] in matter-wave quadratures. Such experiments would open the way to novel tests of quantum mechanics for macroscopic numbers of massive particles.

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