

Geometric Optics using the Vergence Method

Introduction

Geometric optics is the study of the propagation of light in the regime in which diffraction effects (bending of light around corners) are negligible. This is the case when obstacles and openings are much larger than the wavelength of light.

The approach taken in this course is centred around “Vergence Theory”. This is somewhat different from the approach of most textbooks and what you may have learned in the past. Vergences provide a more powerful and elegant approach to solving problems in geometric optics and you will be expected to use this method when confronted with problems of this type.

Vergences describe the curvature of wavefronts. Geometric optics considers the propagation of spherical wavefronts - i.e. wavefronts emanating from point sources. A good two-dimensional analogy is the waves one observes on the surface of a pond after throwing a stone into the water. A ‘vergence’ physically represents the curvature of a wavefront at some given position in space.

Calculating Vergences

The *radius of curvature*, r , of a wavefront is the distance from the source to the current position of the wavefront.

The *curvature* of the wavefront is then given as $1/r$. Near the source, the curvature is very large. A long distance from the source, the wavefront is nearly straight - called a *plane wave*, and the curvature is zero.

A *vergence* is defined as

$$V = \pm n/r \quad (1)$$

where n is the *refractive index* of the medium that the light is propagating through. A sign convention is also introduced:

- for light propagating away from a source (diverging), the vergence is negative i.e. $V = -n/r$
- for light propagating towards a focus (converging), the vergence is positive i.e. $V = +n/r$

The *unit* for vergences is the dioptre, written as D , which is equivalent to m^{-1} .

Common media that you will be working with are air ($n = 1$), water ($n = 1.33$) and glass ($n \sim 1.5$).

Optical Elements

Optical elements such as lenses or mirrors change the curvature of a wavefront. For example, the light from a point source diverges – as the wavefront gets further from the source, the radius of curvature increases and thus the curvature decreases. On passing through a focusing lens, the wavefront is changed so that it is converging – as it propagates the radius of curvature decreases until it reaches zero at the focus.

In terms of vergences, an optical element is said to have a Power, P . The power of a lens is dependent on the shape of the lens and the material that it is made from. Powers also have units of dioptres.

The power of a lens is related to the focal length of the lens, f by

$$P = \pm n/f \quad (2)$$

where n is the refractive index of the medium that the lens is being used in. A lens has a positive power if it is converging and a negative power if it is diverging. In air, the focal length is just the inverse of the power.

Calculating Image Locations

Consider the situation where a lens is used to image an object. For example, you are taking a photograph of a friend. Your friend is the source of scattered light, the lens in your camera is the optical element with a power P , and the film in your camera is where the image is to be formed. A representation of such a situation with source – lens – image is shown in Figure 1. To perform the calculation, depending on the known quantities, we can calculate an initial vergence V_i which relates to the curvature of the wavefront from the source as it hits the lens,

a final vergence V_f which relates to the curvature of the wavefront after it passes through the lens, and P the power of the lens.

The initial vergence (assuming in air, $n = 1$) is

$$V_i = \pm n/r = -1/u \quad (3)$$

Why is this negative? The light from a source diverges – ie the wavefront is expanding and, by convention, this gives a negative vergence. Why is $r = u$? The vergence of the wavefront is evaluated at the lens which is a distance u from the source. Similarly the final vergence is

$$V_f = \pm n/r = 1/v \quad (4)$$

Here we are assuming a real image is formed (it can be viewed on a screen) so the final vergence is positive – the light is converging.

The vergences and the power can be shown to be related by

$$V_f = V_i + P \quad (5)$$

which is known as the vergence equation. Substitution of equations (2), (3) and (4) into equation (5) yields the lens equation that you might be familiar with.

The magnification of the image, M , which is the ratio of the image height to the object height, is given by

$$M = V_i/V_f \quad (6)$$

A negative value for the magnification indicates that the image is inverted.

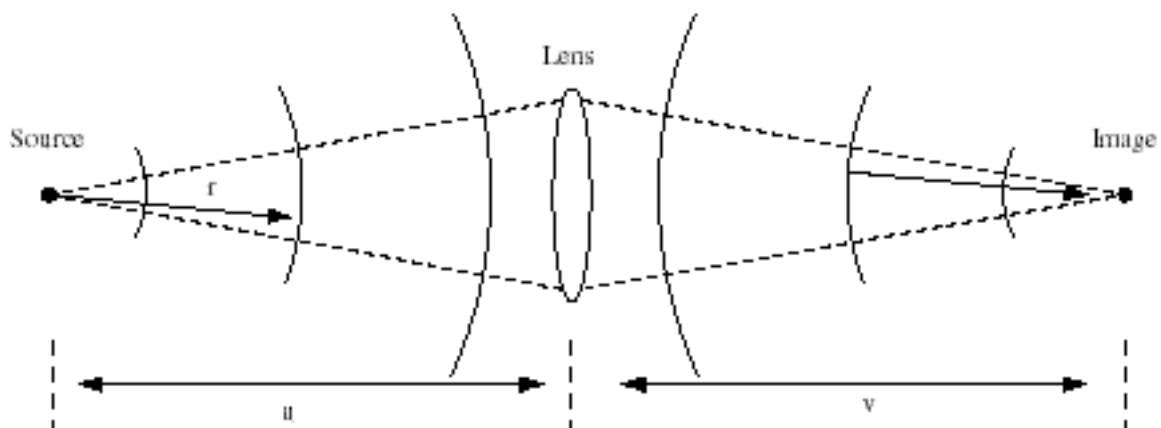


Figure 1. Image formation by a lens

Some Examples

A working understanding of vergences is best obtained by trying some examples. Once you become used to the equations and sign conventions then calculations for lenses, mirrors and interfaces all follow the same simple formula.

A diagram is always useful when solving such problems. A rough sketch can often give a good indication of the answer.

Example 1:

Consider an object in air that is 25 cm from a converging lens with a focal length of 10 cm. Where will the image be formed and what will be its magnification?

The initial vergence is

$$V_i = \pm n/r = -1/0.25 = -4 \text{ D}$$

The power of the lens is

$$P = n/f = +1/0.1 = 10 \text{ D}$$

Equation (5) can be used to determine the final vergence

$$V_f = V_i + P = (-4) + 10 = 6 \text{ D}$$

The image location is found from equation (4)

$$v = 1/V_f = 1/6 = 0.17 \text{ m or } 17 \text{ cm.}$$

As the final vergence is positive, the light is converging and a real image is formed 17 cm from the lens.

From equation (6), the magnification is

$$M = V_i/V_f = -4/6 = -0.66$$

The image is two-thirds the size, and inverted.

Example 2:

Consider the same lens as above but with an object that is only 5 cm from the lens. A summary of the solution is

$$V_i = \pm n/r = -1/0.05 = -20 \text{ D}$$

$$P = n/f = +1/0.1 = 10 \text{ D}$$

$$V_f = V_i + P = (-20) + 10 = -10 \text{ D}$$

$$V_f = \pm n/v = -1/v$$

$$v = -1/V_f = -1/(-10) = 0.1 \text{ m or } 10 \text{ cm.}$$

$$M = V_i/V_f = -20/-10 = +2$$

Here the final vergence is negative, the rays are diverging, and the image is virtual – it is located 10 cm from the lens on the same side as the source. It is twice the size and upright (the magnification is positive).