SUPERCONDUCTING CORRELATIONS IN METALLIC NANO-PARTICLES: WE NEED EXACT SOLUTIONS!

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cond-mat/0106390	BCS model
cond-mat/0110105	Josephson model
nlin.SI/0110049	nucleon model

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Outline

- It is possible to make metallic nano-particles containing a *fixed* number of electrons and measure their *discrete* energy spectrum and see the effect of the *pairing correlations* responsible for bulk superconductivity.

- The pairing correlations are described by the BCS Hamiltonian in the canonical ensemble, which is integrable by the algebraic Bethe ansatz.

- Although the BCS mean-field solution works extremely well for bulk superconductivity it is a poor approximation for small particles with energy level spacing d comparable to the bulk energy gap $\tilde{\Delta}$.

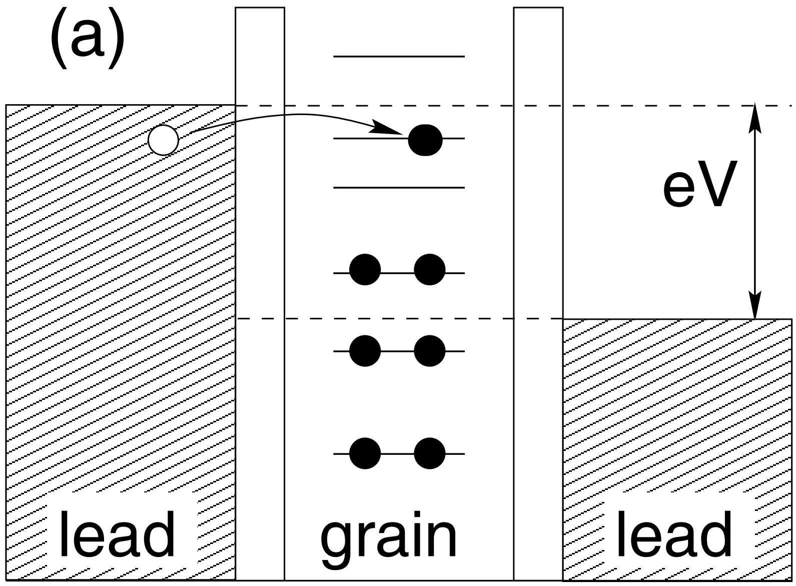
- The exact solution is needed!

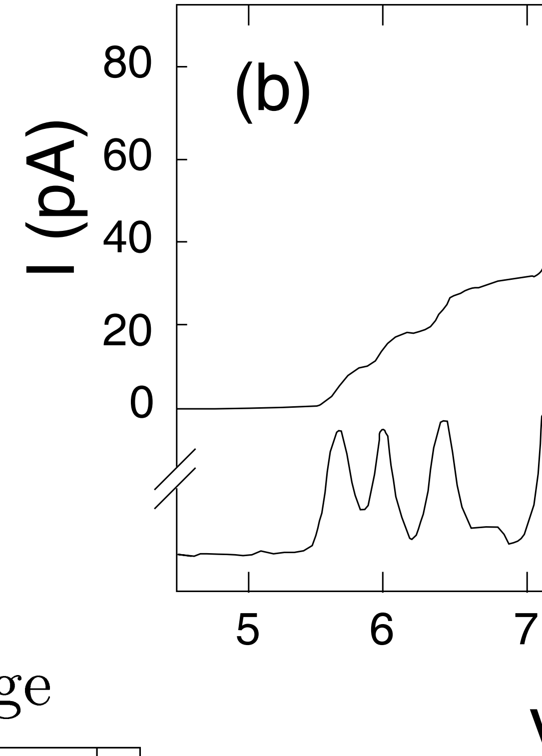
EXPERIMENTS ON METALLIC NANO-PARTICLES

Ralph, Black, and Tinkham, *Phys. Rev. Lett.* **74**, 3241 (1995).

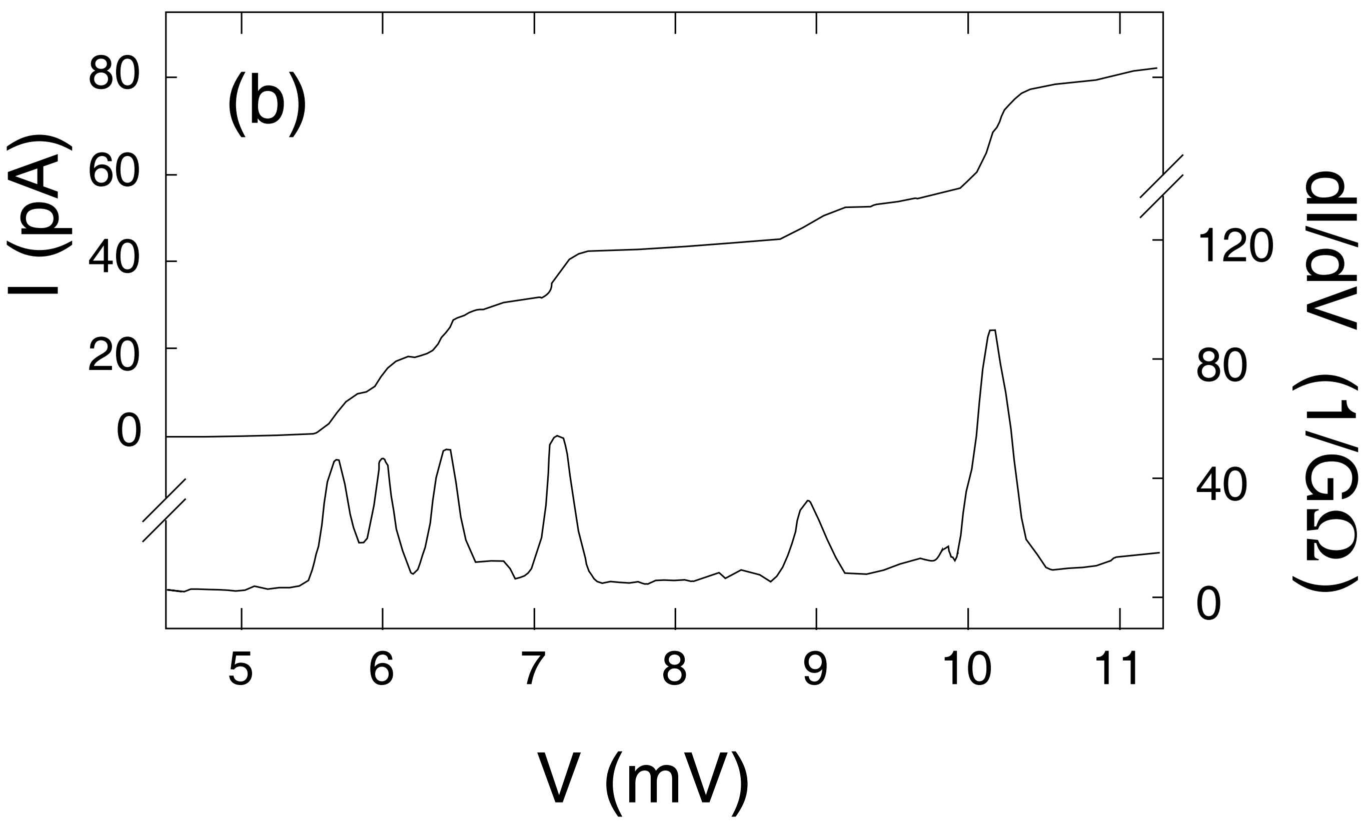
Grains (or nano-particles) of Al are sufficiently *small* (< 10 nm) that one can resolve

- the *discrete* energy spectrum of the electrons
- the addition or removal of a *single* electron. [The grains' charging energy $e^2/2C$ (where C is the capacitance) is sufficiently large that the number N of conduction electrons on the grain is *fixed*. ($N \sim 10^4 - 10^5$).]

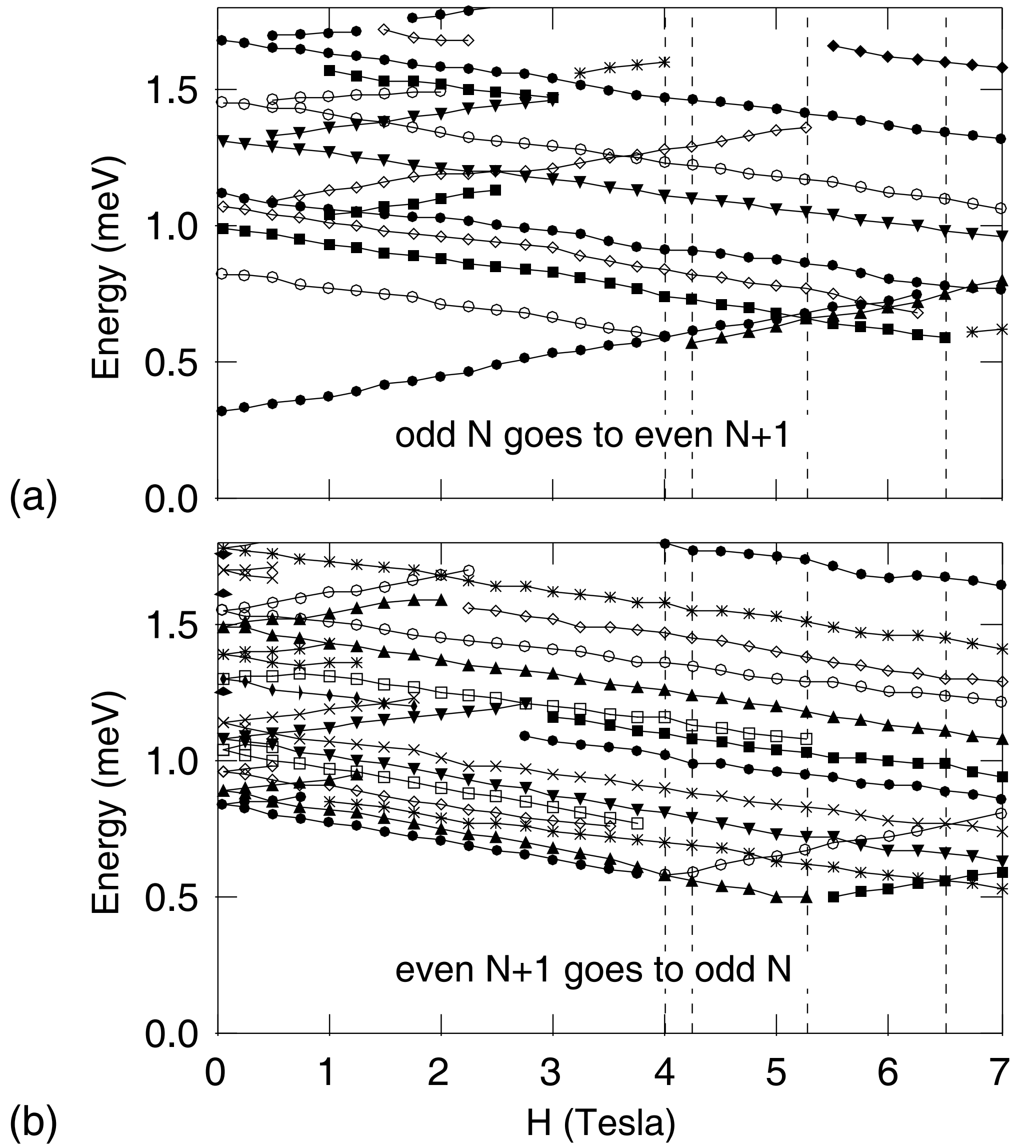




Current versus voltage



Magnetic field dependence of energy levels



Note

- the difference between N odd and even
- the gap due to pairing correlations.

For a review, J. von Delft and D.C. Ralph, *Phys. Rep.*, **345**, 61 (2001).

BCS HAMILTONIAN

Bardeen, Cooper, Schrieffer (1957)

$$H = \sum_{j,\sigma=\pm} (\varepsilon_j - \mu - \sigma h) c^\dagger_{j\sigma} c_{j\sigma}$$

$$-\lambda d \sum_{ij,|\varepsilon_j|,\varepsilon_i|<\omega_D} c^\dagger_{i+} c^\dagger_{i-} c_{j-} c_{j+}$$

$\pm$ denotes the electron spin.

$c^\dagger_{j\sigma}$ is the fermion creation operator for level j with spin σ and energy ε_j.

λ is the dimensionless coupling constant.

d is the energy level spacing.

$-\sigma h \equiv \frac{1}{2}\sigma \mu_B g B$ is the Zeeman energy of a spin σ electron in a magnetic field B.

Pairing only occurs for the $\Omega \equiv 2\omega_D/d$ pairs (~ 150) of electrons within an energy ω_D of the Fermi energy, where ω_D is the Debye frequency.

MEAN-FIELD THEORY

Grand canonical ensemble (variable N)

$$|BCS\rangle = \prod_j (u_j + e^{i\phi_j} v_j c^\dagger_{j+} c^\dagger_{j-})\,|Vac\rangle$$

with $u_j^2 + v_j^2 = 1$, where the variational parameters u_j and v_j are real.
$|BCS\rangle$ is not an eigenstate of the fermion number operator $\hat{N} \equiv \sum_{j,\sigma} c^\dagger_{j,\sigma} c_{j,\sigma}$. Particle number is fixed only on the average by the condition $\langle \hat{N} \rangle_{BCS} = N$, which determines the chemical potential μ.

$$\Delta_{MF} \equiv \lambda\, d \sum_j \langle c_{j+} c_{j-} \rangle_{BCS} = \lambda\, d \sum_j u_j v_j e^{i\phi_j}$$

Gauge symmetry is broken.
A self-consistent solution leads to a pairing parameter and energy gap

$$\tilde{\Delta} = \omega_D / \sinh(1/\lambda).$$

Predicts superconductivity disappears at finite $d/\tilde{\Delta}$.

The BCS mean-field theory works *extremely* well for conventional *bulk* superconductors.

Energy gap versus temperature

$$\Delta(T = 0) = 1.76 k_B T_c$$

CANONICAL ENSEMBLE

N is fixed, and so the ground state is an eigenstate of the fermion number operator, $\hat{N}$, implying

$$\langle c_{j+} c_{j-} \rangle = 0$$

Hence, the traditional superconducting order parameter will vanish and gauge symmetry is not broken.

What signatures of superconductivity or Cooper pairing persist?
-energy gap?
-condensation energy?
-parity dependence of energy gap?

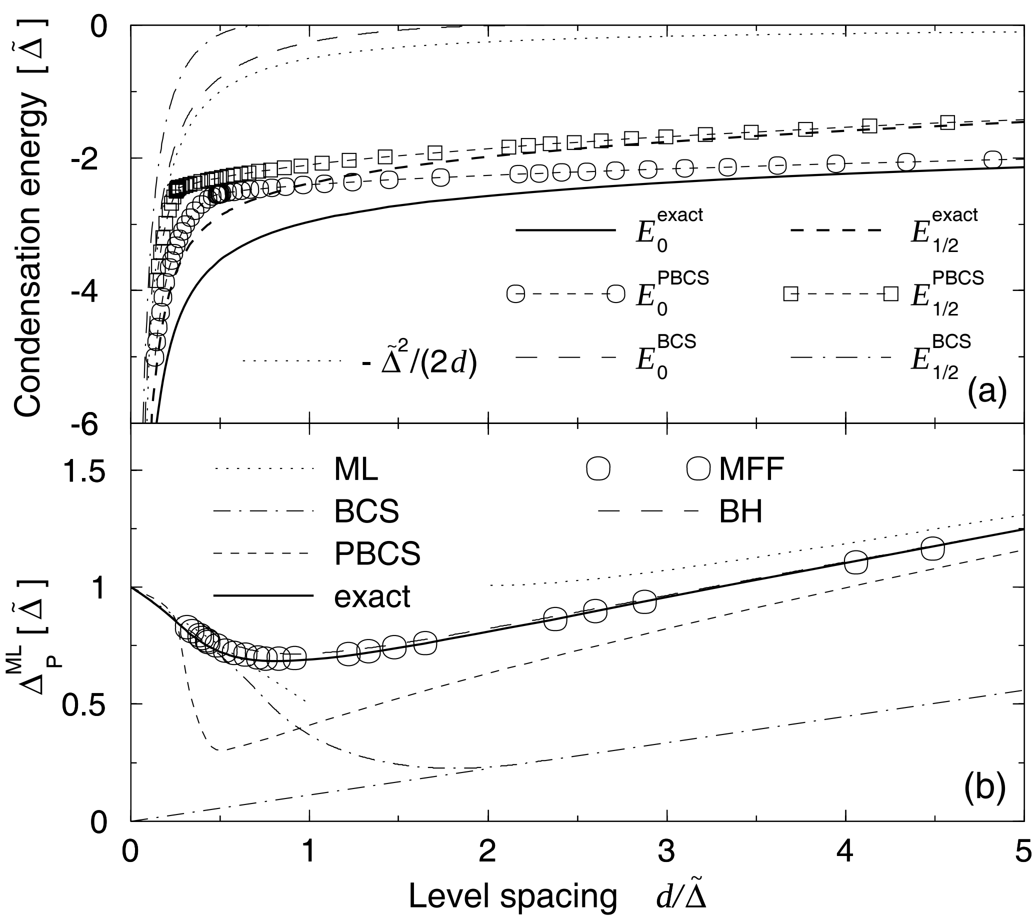
The model is integrable by the Bethe ansatz.

COMPARISON WITH APPROXIMATIONS

- *Condensation energy* is the energy of the ground state relative to the expectation value of the Hamiltonian in an uncorrelated state.
- Matveev and Larkin's parity parameter

$$\Delta_P^{ML} \equiv E_{1/2}^{N+1} - \frac{1}{2}(E_0^N + E_0^{N+2})$$

where N is even.



CORRELATION FUNCTIONS

$$D_{ij} \equiv \langle c^\dagger_{i+} c^\dagger_{i-} c_{j+} c_{j-} \rangle - \langle c^\dagger_{i+} c_{j+} \rangle \langle c^\dagger_{i-} c_{j-} \rangle$$

We have evaluated this exactly using the quantum inverse scattering method.

In the bulk limit the canonical pairing parameter $\Delta_{\rm can}$ given by

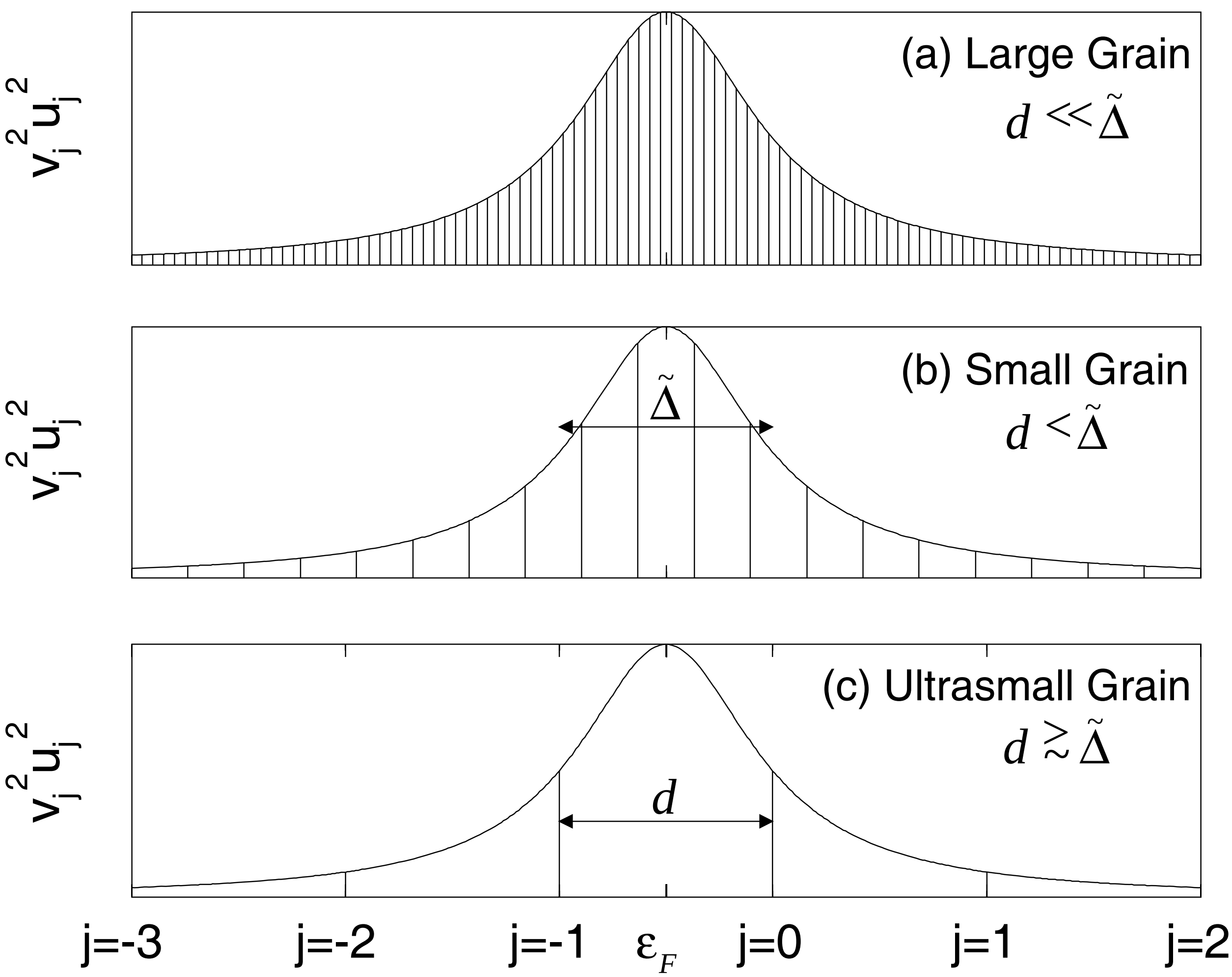
$$\Delta^2_{\rm can} \equiv (\lambda\, d)^2 \sum_{ij} D_{ij}$$

reduces to $\tilde{\Delta}$.

$C^2_j(d) \equiv D_{jj}$ corresponds to the probability enhancement for finding a pair of electrons instead of two uncorrelated electrons in a single particle level.

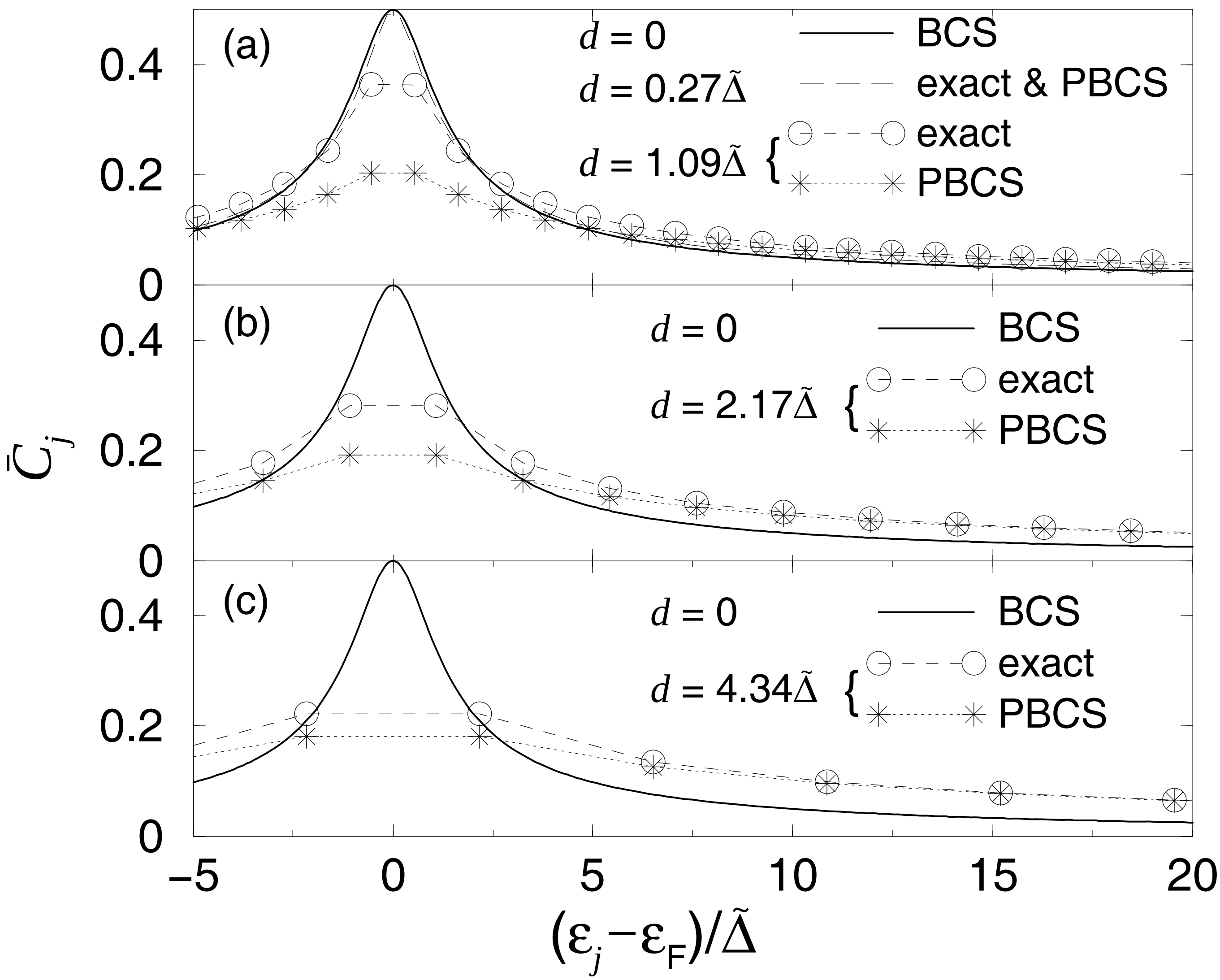
Why does superconductivity break down (according to the grand canonical ensemble) when $d \sim \tilde{\Delta}$?

$$C_j^2(d)_{BCS} = v_j^2 u_j^2$$



$\tilde{\Delta}/d$ is roughly the number of Cooper pairs in the system.

Correlations near the Fermi energy
Braun and von Delft



For $d \sim \Delta$ the projected BCS becomes unreliable

SUMMARY

In the canonical ensemble (fixed N) pairing correlations do persist for arbitrarily large d/Δ. i.e., the condensation energy does not vanish.

There is a smooth crossover between two regimes:

(i) $d \ll \Delta$ Bulk superconducting regime
* Condensation energy is extensive ($\propto 1/d$).
* Pairing correlations are strongly localised around the Fermi energy.

(ii) $d \gg \Delta$ Fluctuation-dominated regime
* Condensation energy is intensive (independent of d).
* Pairing correlations are spread out in energy.

CONCLUSIONS

- It is possible to make metallic nanoparticles containing a *fixed* number of electrons and measure their *discrete* energy spectrum and see the effect of the pairing correlations responsible for bulk superconductivity.
- These pairing correlations are described by the BCS Hamiltonian, which is integrable by the algebraic Bethe ansatz.
- Although the BCS mean-field solution works extremely well for bulk superconductivity it is a poor approximation for small particles with $d > \tilde{\Delta}$. The exact solution is needed!
- Important open questions:
  * field dependence of the correlations?
  * exact solution for d-wave pairing?
  * exact solution for finite temperature?
  * Josephson coupling between pairs of grains?
- This field presents tremendous opportunities for the integrable models community!