PHYS2100 Assignment

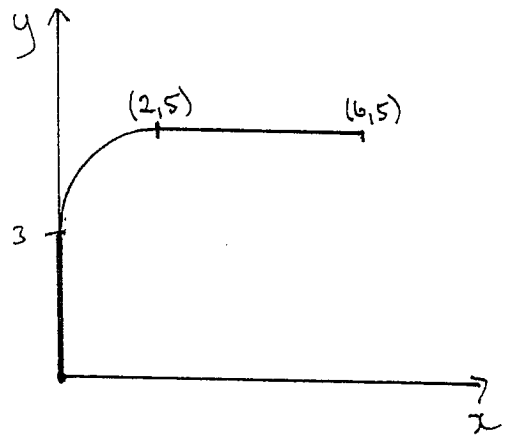
Due 5pm Friday 2nd September, 2006

1. (i) Determine whether the following forces are conservative, and if so, then determine their potential functions:

   (a) $\mathbf{F_a} = (2y+1)z\,\hat{\mathbf{i}} + (2xz + z + y^2)\,\hat{\mathbf{j}} + (2xy + x + y)\,\hat{\mathbf{k}}$.

   (b) $\mathbf{F_b} = 3y\,\hat{\mathbf{i}} + (1-3x)\,\hat{\mathbf{j}}$.

   (ii) A wire lies along the path shown in the diagram below. It consists of a straight section, followed by a quarter turn of constant radius, followed by another straight section. The force $\mathbf{F_b}$ given above pushes a bead along the wire from the origin to the point $(6, 5)$. Calculate the work done by $\mathbf{F_b}$ in moving the bead.

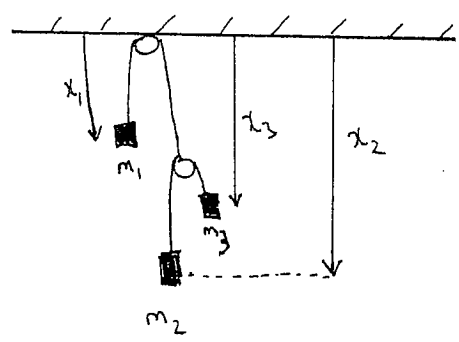


2. A particle of mass m is attracted towards the origin by a force of magnitude k^2/x^3.

   (i) Show the force is conservative, and find the corresponding potential energy.

   (ii) Assuming the particle starts at rest a distance d away from the origin, show that the time it takes to reach the origin is $\frac{d^2\sqrt{m}}{k}$.

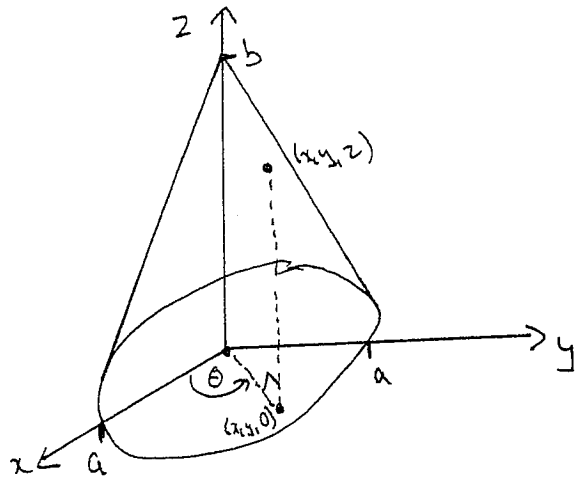
3. Consider the pulley system shown.
   Ignoring friction and the mass of the pulleys, show that the acceleration of mass m_1 is given by



$$\ddot{x_1} = \left[\frac{m_1(m_2+m_3) - 4m_2m_3}{m_1(m_2+m_3) + 4m_2m_3}\right] g$$

   where g is the acceleration due to gravity.

4. A smooth right-circular cone of base radius a and perpendicular height b stands with its base centred on the origin and its apex at point $(0, 0, b)$. Find Lagrange's equations of motion for a particle of mass m constrained to move on the cone, using the angle θ in the $x - y$ plane and the z-coordinate as the generalised coordinates.



5. Consider a simple pendulum consisting of a mass m attached to a string of length l. When the pendulum is set into motion, the length of the string is shortened at a constant rate α, i.e.

$$\frac{dl}{dt} = -\alpha.$$

(i) Compute the Lagrangian L and the Hamiltonian H of the system.

(ii) Is the Hamiltonian equal to the total energy E? Which of E, H are constant? Explain these results.