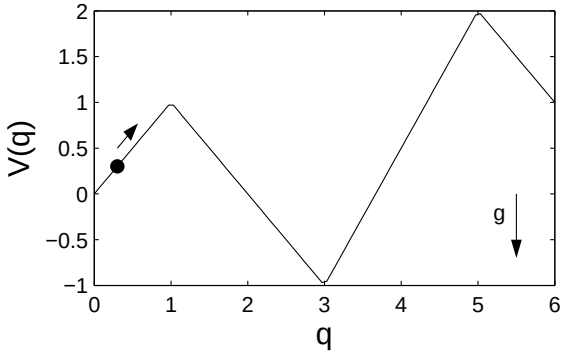
PHYS2100: Hamiltonian mechanics tutorial sheet 1
Problem 5: Due 5pm Friday 6th October 2006.

** $\equiv$ To be handed in.

1. Consider a bead on a frictionless wire under the influence of gravity as illustrated on the right. Assume that the bead has sufficient energy to travel from the initial position at $q = 0$ to the final position $q = 6$. Plot the *shape* of the curve in phase space from $0 \leq q \leq 6$. (i.e. focus on the shape and position of peaks — not the absolute values of momentum).



2. Consider the 1D system with the Hamiltonian

$$H = \frac{p^2}{2m} + k|q|.$$

Draw the vector flow field on a phase space diagram for m=2 and k=0.5 in the region $-2 \leq p \leq 2$, $-2 \leq p \leq 2$.

3. Prove that the phase space of the following Hamiltonian has no fixed points

$$H = \frac{p^2}{2m} + aq, \qquad a > 0.$$

4. For the linear repulsive force Hamiltonian

$$H = \frac{p^2}{2m} - \frac{1}{2}aq^2, \qquad a > 0$$

show that the general solution is

$$q(t) = A_1 e^{\gamma t} + A_2 e^{-\gamma t}, \qquad \gamma = \sqrt{a/m},$$

and hence that

$$H = -2A_1 A_2.$$

5**. A particle of mass $m = 1/2$ moves in the Duffing potential describing a double well

$$V(q) = q^4 - \frac{q^2}{2}.$$

Sketch the phase portrait and find the fixed points, if any exist. If any hyperbolic fixed points exist, find the equation of the separatrices.