PHYS2100: Hamiltonian mechanics tutorial sheet 2
Due 5pm Friday 13th October 2006.

** $\equiv$ To be handed in.

1. Prove that for any function $f(q, p, t)$ then

$$\dot{f} = \{f, H\} + \frac{\partial f}{\partial t}.$$

2. Show that if $\{Q, P\}_{(q,p)} = 1$ then

$$\dot{P} = -\frac{\partial \bar{H}}{\partial Q}$$

where $\bar{H}(Q, P) = H(q, p)$.

3. Show that the following transformation is canonical

$$\begin{aligned} Q &= e^{\lambda}(q\cos\theta + p\sin\theta), \\ P &= e^{-\lambda}(-q\sin\theta + p\cos\theta). \end{aligned}$$

4. Show that the area enclosed by the separatrix of the vertical pendulum with Hamiltonian

$$H = \frac{l^2}{2} - \alpha^2\cos\theta,$$

is 16α. Deduce that the maximum value of the action for librating motion is $8\alpha/\pi$.

5**. A particle of mass m experiences the potential

$$\begin{aligned} V(\psi) &= A\psi, \qquad (0 \le \psi \le \alpha), \\ V(\psi) &= A\alpha, \qquad (\alpha \le \psi \le \pi), \\ V(\psi) &= V(-\psi), \end{aligned}$$

which is defined to be periodic outside the range $(-\pi \le \psi \le \pi)$.

(a) Sketch a graph of this potential, and draw the phase portrait for the system. What is the energy that separates the librations and rotations?

(b) Find the action-angle variables for each type of motion, and determine the frequency ω of motion for a trajectory of energy E. Sketch a graph of the frequency ω versus E.