Questions 2 and 7 are due Fri 4th August by 5pm.

PHYS2100 Problem Sheet 1

Semester 2, 2006

Problems marked † are harder than the rest.

1. A particle moves towards a centre of attraction starting from rest a distance a from the centre. If its velocity at a distance x from the centre varies as (α a constant)

$$\alpha\Big(\frac{a^2-x^2}{x^2}\Big)^{1/2},$$

determine x as a function of t.

2. A particle of mass m is acted on by a force

$$m\mu(x+a^4/x^3)$$

towards the origin. Here x denotes the distance of the particle from the origin and μ, a are positive constants.

(i) Show that the motion is conservative and find the corresponding potential energy.

(ii) Assuming that the particle starts at rest a distance a from the origin, show that the particle's velocity is given by

$$v=-\sqrt{\mu}\Big(\frac{a^4-x^4}{x^2}\Big)^{1/2}.$$

(iii)† Show that the particle will arrive at the origin at time $\pi/(4\sqrt{\mu})$.

3. Calculate the escape velocity from earth, given that the radius of the earth is 6.378 $\times$ 10^6m.

4. A star is attracted along a straight line towards a black hole with Schwarzild radius R. Given that the velocity of the star at an infinite distance from the black hole is zero, show that the velocity of the star varies with its distance x from the centre of the black hole according to

$$v=-c\sqrt{\frac{R}{x}}$$

where c is the speed of light.

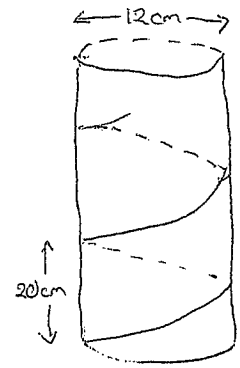
5. Show that a particle in 2-space which moves so that it its position vector is always orthogonal to its velocity vector, must move in a circular path.

6. Find the length of arc for the following curves between $t = 0$ and $t = 1$:

   (i) $(x, y, z) = (\frac{1}{2}t, \frac{1}{3}(1-t)^{\frac{3}{2}}, \frac{1}{3}(1+t)^{\frac{3}{2}})$

   (ii) $(x, y, z) = (t\sin t + \cos t, \sin t - t\cos t, \frac{1}{3}t^3)$.

7. Copper tubing is wrapped in a circular helix around a cylindrical core that has a 12 cm diameter. What length of tubing will make one complete turn around the cylinder in a distance of 20cm measured along the axis of the cylinder?



8. Determine the work done by the gravitational force

$$F = -mg\hat{\mathbf{k}}$$

   on a particle with mass m which moves along the expanding helix

$$(x, y, z) = (t\cos t, \sin t, t), \qquad 0 \le t \le \frac{\pi}{2}.$$

9. Let $\mathbf{a}(t), \mathbf{b}(t)$ be vector functions in 3-space and $f(t)$ a scalar function of t. Prove the following:

   (i) $\frac{d}{dt}(f(t)\mathbf{a}) = \dot{f}\mathbf{a} + f\dot{\mathbf{a}}$

   (ii) $\frac{d}{dt}(\mathbf{a}\cdot\mathbf{b}) = \dot{\mathbf{a}}\cdot\mathbf{b} + \mathbf{a}\cdot\dot{\mathbf{b}}$

   (iii) $\frac{d}{dt}(\mathbf{a}\times\mathbf{b}) = \dot{\mathbf{a}}\times\mathbf{b} + \mathbf{a}\times\dot{\mathbf{b}}$.