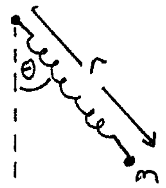
Questions 3 and 5 due Friday 25th August.

PHYS2100 Problem Sheet 4

Semester 2, 2006

1. A particle of mass m is suspended by a spring from a fixed point and constrained to move in a vertical plane containing the point of support. Using r, θ as the generalised coordinates shown, obtain the corresponding generalised forces and show that the system is conservative. Hence derive the equations of motion of the system.



2. A light wire bent in the shape of a circle rotates freely about a vertical diameter. A bead of mass m slides without friction on the wire. Use as generalised coordinates the angle between the vertical line and the radial line from the centre of the circle to the bead and the angle of rotation of the plane of the wire. Check that the total energy is constant and show that the angular momentum about the vertical diameter is also constant.

3. Two blocks are connected by a light inextensible cord. One block of mass m is placed on a smooth horizontal table, the other block of mass M hangs over the edge of the table, the string passing over a frictionless pulley. Find the acceleration of the system. What would you expect the acceleration to be in the limit $m \to 0$? $M \to 0$? Does your expression for the acceleration satisfy these limits?

4. A smooth straight wire is constrained to rotate in a horizontal plane with constant angular velocity ω about a vertical axis through one end. The point on the wire at a distance b from the axis is the centre of an attractive force proportional to the distance with proportionality constant k. A bead of mass m is free to slide on the wire. Find the position of the bead as a function of time. Take the case $m\omega^2 < k$. Is this oscillatory?

5. A spherical pendulum of length a is held out horizontally, and the bob is started with a horizontal velocity v_0. Show that it falls to a depth below its original position given by

$$\frac{v_0^2}{4g}\left[\left(1+\frac{16a^2g^2}{v_0^4}\right)^{\frac{1}{2}}-1\right] \approx \frac{2ga^2}{v_0^2}, \quad \text{for } \frac{ag}{v_0^2} \ll 1.$$