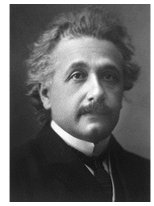
Relativity

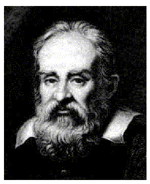
Isaac Newton
1642-1727

Hendrik A. Lorentz
July 18, 1853 -February 4, 1928

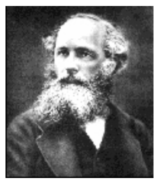
Henri Poincaré
April 29, 1854 - July 17, 1912



Albert Einstein
March 14, 1879 -April 18, 1955

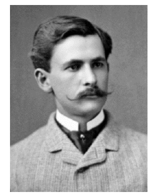


Galileo
1564 -1642



James Maxwell
1831-1879

Hermann Minkowski
22 June, 1864 -12 Jan, 1909



Albert Abraham Michelson
December 19, 1852-May 9, 1931

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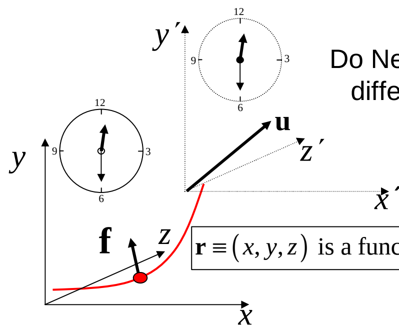
Lecture 1

Newtonian relativity



$\mathbf{r} \equiv (x, y, z)$ is a function of t

Do Newton's equations of motion hold true in a different *non accelerating* reference frame?

$$x' = x - u_x t$$
$$y' = y - u_y t$$
$$z' = z - u_z t$$
$$t' = t$$

Galilean transformations

$$m\frac{d^2\mathbf{r}(t)}{dt^2} = \vec{\mathbf{f}}$$ Newton's law

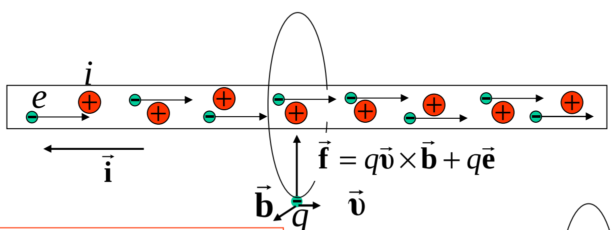
Acceleration of a particle is the same in both frames!

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{d^2\mathbf{r}'}{dt'^2}$$

Conclussion: Laws of mechanics are identical in all inertial systems

This looks OK but there is a catch!
We have assumed that **f** = **f′**

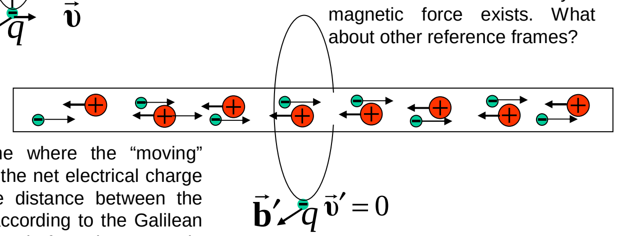
Forces between a moving electrical charge and an electrical current



$$\vec{\mathbf{f}} = q\vec{\boldsymbol{\upsilon}} \times \vec{\mathbf{b}} + q\vec{\mathbf{e}}$$

The wire has zero net electrical charge because the number of electrons per 1 meter of wire equals the number of ions. Therefore the total electrical field **e** acting on the moving electron is zero and only a magnetic force exists. What about other reference frames?

Note that for a reason which will become clear in Lecture 2, we use in this course low case **bold** characters for 3-dimensional vectors.



$$\vec{\boldsymbol{\upsilon}}' = 0$$

$$\vec{\mathbf{f}}' = q\vec{\boldsymbol{\upsilon}}' \times \vec{\mathbf{b}}' + q\vec{\mathbf{e}}' = 0!?$$

In a moving inertial reference frame where the "moving" electron is at rest, one can argue that the net electrical charge of the wire is still zero because the distance between the electrons and ions does not change according to the Galilean transformations. Therefore the electrostatic force is zero as in the "stationary" frame. The magnetic force is also zero because there is no magnetic force if the charge is not moving. We may conclude that there is no force on the electron and therefore the electron does not accelerate. But there is an acceleration in the "stationary" inertial frame! Something must be wrong!

The speed of EM wave is *c*. But in which reference frame?

Does the wave equation hold true in all inertial frames?
If we take a wave equation and apply Galilean transformations a problem arises.

$$\boxed{c^2\frac{\partial^2\psi(x,t)}{\partial x^2}-\frac{\partial^2\psi(x,t)}{\partial t^2}=0} \longrightarrow \boxed{\begin{aligned}x'&=x-ut\\ t'&=t\end{aligned}}$$

See Appendix 1 for details

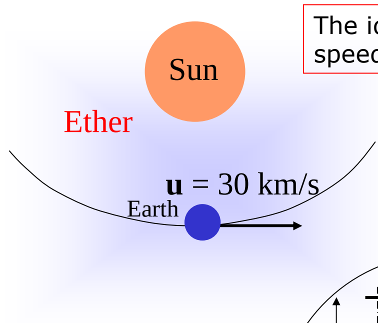
$$\boxed{\left(c^2+u^2\right)\frac{\partial^2\psi(x',t')}{\partial x'^2}-\frac{\partial^2\psi(x',t')}{\partial t'^2}-2u\frac{\partial^2\psi(x',t')}{\partial x'\partial t'}=0}$$

In which reference frame does the wave equation take its conventional form?

This question is easy to answer for other waves. For example, sound waves or waves on the surface of water have obvious preferred reference frames. These are the frames where the two media in which the waves propagate do not move as a whole. Electromagnetic waves propagate in a vacuum (apparently no propagation medium). Is the vacuum not empty? May be the vacuum is filled with Ether.

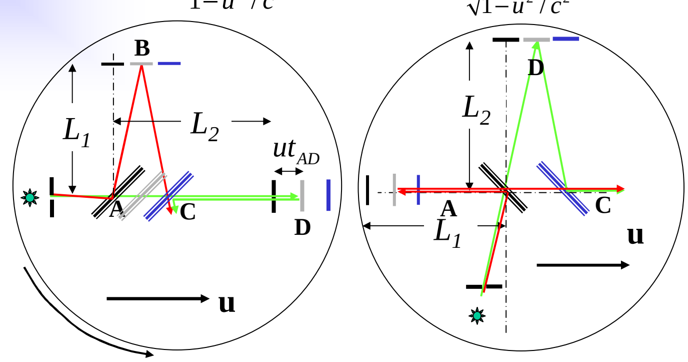
Ether and Michelson – Morley experiment

The idea of this experiment was to measure the speed of the Earth relative to the ether.



$$t_{ABC}=\frac{2L_1/c}{\sqrt{1-u^2/c^2}} \qquad t_{ABC}=\frac{2L_1/c}{1-u^2/c^2}$$

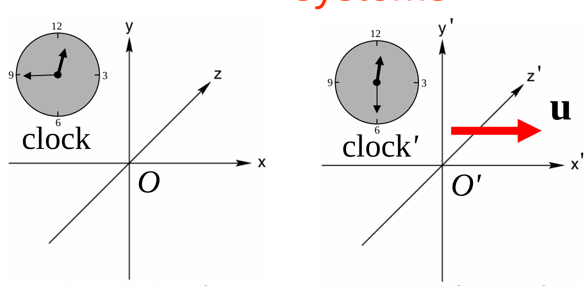
$$t_{ADC}=\frac{2L_2/c}{1-u^2/c^2} \qquad t_{ADC}=\frac{2L_2/c}{\sqrt{1-u^2/c^2}}$$



Lorentz transformations

Lorentz transformations provide a systematic approach to transformation from one reference frame to another of a physical event space location and its time. No internal contradiction can arise as a result of postulating this transformation (as well as Galilean transformation). Only an *experiment* (a real one but not a "thought experiment") can test the validity of Lorentz transformations. Up to date all experiments within their accuracy are in agreement with Lorentz transformations.

Standard set of reference frames used in this course. Describing events in two coordinate systems



Each clock is at rest in the corresponding reference frame.

Events are described by four numbers in each reference frame: x,y,z, and t in O and x',y',z',and t' in O'. Event coordinates will be written as (x,y,z,t) or (x_1,x_2,x_3,t) or (x_1,x_2,x_3,x_4).

Conventionally, when the two origins O and O' coincide, the two clocks are assumed to be set to zero.

If the speed of light is formally set to infinity, the Lorentz transformations should be equivalent to the Galilean transformations.

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Einstein's postulates

Einstein formulated his version of special relativity in 1905 after Lorentz-Poincare theory was published

Einstein's postulated

1. Absolute uniform motion cannot be detected
2. The speed of light is the same for all observers

Then he derived the Lorentz transformation

There is no ether in the Einstein's model. Electromagnetic waves propagate in vacuum. Instead of talking about properties of ether he is talking about properties of space and time. The two theories (by Lorentz-Poincare and by Einstein) are mathematically equivalent. It is more or less a philosophical (not mathematical) question what to postulate – directly Lorentz transformations or the constancy of the speed of light.

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Deriving the Lorentz transformations form the Einstein's postulates

Because space-time is homogeneous and isotropic the transformations should be linear. This is why.

Take a particle moving with a constant velocity and sending signals with a constant rate (see Fig.). The time intervals between the signals as measured by *Clock I* should not change as the particle continues its motion. This is also true for *Clock II*. Therefore $dt/dt' = A,$ where A is a constant. We also demand that if a particle moves with a time independent velocity in one inertial frame, then this particle should have a time independent velocity in any other inertial reference frame.

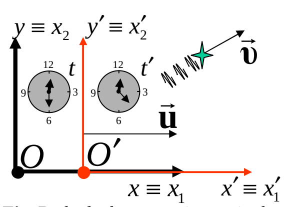

Fig. Both clocks are stationary in the corresponding reference frames.

Assume that there are three function g_1, g_2, and g_3 which relate coordinates of the moving object in the primed reference frame to coordinates and time of this object in the not primed reference frame

$$x'_i = g_i\left(x_1, x_2, x_3, t\right), \quad i = 1,2,3$$

Because $\upsilon'_i \equiv \dfrac{dx'_i}{dt'} = \dfrac{\partial g_i}{\partial t}\dfrac{dt}{dt'} + \sum_{k=1}^{3}\dfrac{\partial g_i}{\partial x_k}\dfrac{dx_k}{dt}\dfrac{dt}{dt'} = A\left(\dfrac{\partial g_i}{\partial t} + \sum_k \upsilon_k \dfrac{\partial g_i}{\partial x_k}\right)$ should be a constant for any constant values of υ_k, each partial derivative should be a constant. Therefore the functions g are linear functions of x_1, x_2, x_3, and t.

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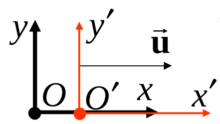
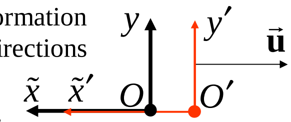
Point O' has coordinates (0,0,0) in the $x'y'z'$ reference frame and (ut, 0,0) in the xyz reference frame therefore the most general linear transformation reads

$$x' = \gamma(x - ut) + \alpha y + \beta z,$$



$$x' = \gamma(u)(x - ut)$$

where α = β = 0 because this transformation should not change if we reverse the directions of y and/or z axis (space is isotropic). If we reverse the directions of the x-axes, this changes the sign of the velocity u and the transformation for the new pair of reference frames can be written as $\tilde{x}' = \gamma(-u)(\tilde{x} + ut)$

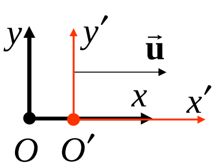


$$\tilde{x}' = \gamma(-u)(\tilde{x} + ut)$$

On the other hand, the relation between the old coordinates and new is $\tilde{x}' = -x'$

and consequently $-x' = \gamma(-u)(-x + ut)$ or $x' = \gamma(-u)(x - ut)$

Therefore $\boxed{\gamma(u) = \gamma(-u)}$



Because the two reference frames are can be treaded equivalently, transformation from reference frame O' to frame O can be obtained by replacing u with -u .

$$x = \gamma(-u)(x' + ut') = \gamma(u)(x' + ut')$$

Because the speed of light is the same in the two reference frames, the following two equations $x' = ct'$ and $x = ct$ should hold simultaneously.

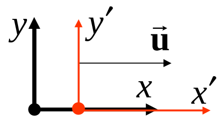
Substituting these equations into $x = \gamma(x' + ut')$ and $x' = \gamma(x - ut)$ we obtain

$$ct = \gamma(ct' + ut') = \gamma t'(c + u) \quad \text{and} \quad ct' = \gamma(ct - ut) = \gamma t(c - u)$$

We multiply these two equations to obtain an equation for γ

$$c^2 t't = \gamma^2 tt'(c - u)(c + u) = \gamma^2 tt'(c^2 - u^2)$$

$$\boxed{\gamma = \sqrt{\frac{c^2}{c^2 - u^2}} = \frac{1}{\sqrt{1 - u^2/c^2}}}$$



$$\boxed{x' = \gamma\left(x - ut\right)}$$

$$\boxed{x = \gamma\left(x' + ut'\right)}$$

We substitute the first equation into the second and solve it for t'

$$x = \gamma\left(\gamma\left(x - ut\right) + ut'\right) = \gamma^2 x - \gamma^2 ut + \gamma ut'$$

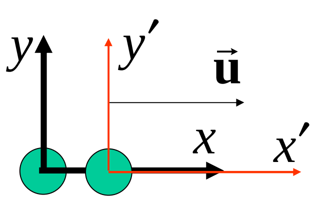
$$t' = \gamma t + \frac{1-\gamma^2}{\gamma u} x = \gamma t + \frac{1 - \dfrac{1}{1-u^2/c^2}}{\gamma u} x = \gamma\left(t - \frac{u}{c^2}x\right)$$

$$\boxed{t' = \gamma\left(t - \frac{u}{c^2}x\right)} \qquad \boxed{t = \gamma\left(t' + \frac{u}{c^2}x'\right)}$$

Sometimes the sign in the equations may be confusing. Remember that u, t, and x on one side are referred to the same frame. The velocity simply changes sign in the other frame

Lorentz transformations (summary)

For two reference frames whose clocks are set to zero when their origins coincide



$$\boxed{\begin{aligned} x' &= \gamma\left(x - ut\right) \\ y' &= y; \quad z' = z \\ t' &= \gamma\left(t - \frac{u}{c^2}x\right) \end{aligned}}$$

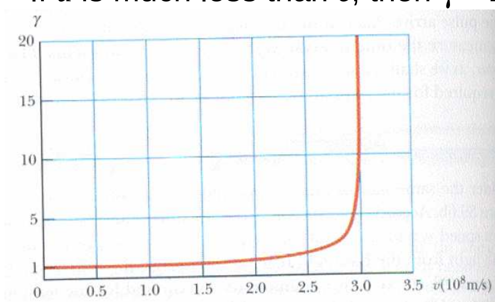
Gamma

A parameter that appears a lot in relativity

$$\boxed{\gamma(u) \equiv \frac{1}{\sqrt{1-u^2/c^2}}}$$

Gamma is always greater than 1

If u is much less than c, then $\gamma \approx 1$



Approximate Values for γ at Various Speeds

v/c	γ
0.001 0	1.000 000 5
0.010	1.000 05
0.10	1.005
0.20	1.021
0.30	1.048
0.40	1.091
0.50	1.155
0.60	1.250
0.70	1.400
0.80	1.667
0.90	2.294
0.92	2.552
0.94	2.931
0.96	3.571
0.98	5.025
0.99	7.089
0.995	10.01
0.999	22.37

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Interval

An square of an interval Δs between two events (x_1,t_1) and (x_2,t_2) is defined by the expression $\boxed{\Delta s^2 = c^2\Delta t^2 - \Delta x^2 \equiv c^2(t_2-t_1)^2 - (x_2-x_1)^2}$

Now we calculate Δs^2 in two reference frames when time and spatial coordinates obey Lorentz's transformations: $t = \gamma\left(ux'/c^2 + t'\right)$ and $x = \gamma\left(x' + ut'\right)$

$$c^2\Delta t^2 - \Delta x^2 = c^2\gamma^2\left(\Delta t' + u\Delta x'/c^2\right)^2 - \gamma^2\left(u\Delta t' + \Delta x'\right)^2 =$$

$$= \gamma^2 c^2 \Delta t'^2 + \gamma^2\frac{u^2}{c^2}\Delta x'^2 + 2\gamma^2 u\Delta t'\Delta x' - \gamma^2 u^2\Delta t'^2 - \gamma^2\Delta x'^2 - 2\gamma^2 u\Delta t'\Delta x' =$$

$$\gamma^2\left(c^2-u^2\right)\Delta t'^2 - \gamma^2\left(1-\frac{u^2}{c^2}\right)\Delta x'^2 = \Delta t'^2 - \Delta x'^2$$

$$\boxed{c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2 - \Delta x'^2}$$

Generally: $\boxed{c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = c^2\Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2}$

Interval does not change (it is an invariant) under Lorentz transformation

Note 1: Classical length is *not* invariant under Lorentz transformation

Note 2: The interval is also invariant under rotation in ordinary 3D space

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Lecture 2

Proper time

A proper time interval is defined by the relation

$$\boxed{\Delta\tau^2 = \frac{\Delta s^2}{c^2}}$$

1. Obviously the proper time is an invariant under Lorentz transformations.
2. This time corresponds to the time shown by a clock which does not move.

$$\Delta\tau^2 \equiv \frac{\Delta s^2}{c^2} \equiv \Delta t^2 - \frac{\Delta x^2 + \Delta y^2 + \Delta z^2}{c^2} = \Delta t^2$$

Because if the clock does not move $\Delta x^2 + \Delta y^2 + \Delta z^2 = 0$

3. In the reference frame where the clock is moving $\Delta t = \gamma\Delta\tau$

Generally $\Delta t = \gamma\left(u\Delta x'/c^2 + \Delta t'\right)$

But for a clock measuring the proper time $\Delta x' \equiv 0$ and $\Delta t' \equiv \Delta\tau$

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Absolute future and absolute past

If Δs^2 is larger than zero in one reference frame it is larger than zero in all other inertial reference frames.

$\boxed{\Delta s^2 > 0}$ Note: in Newton's mechanics c is infinitely large and this inequality always holds.

Two such events can have the same location in a specially selected reference frame

Lorentz transformations relate "new" and "old" displacements $\Delta x' = \gamma(\Delta x - u\Delta t)$

Therefore $\Delta x' = 0$ if $u = \Delta x / \Delta t$

Because $|u|$ must be smaller than c one gets $\Delta x^2 / \Delta t^2 < c^2$, that is $\Delta s^2 > 0$

There is absolute future and absolute past for such events!

If $\left\{c^2\Delta t^2 - \Delta x^2 > 0\right\} \to \left\{\frac{|\Delta x|}{c} < |\Delta t|\right\} \to \left\{\left|\frac{u}{c}\frac{\Delta x}{c}\right| < |\Delta t|\right\}$ remember that $|u| < c$

Therefore sign of $\Delta t' = \gamma\left(\Delta t - \frac{u}{c^2}\Delta x\right)$ coincides with sign of Δt.

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Relativity of simultaneity

$\boxed{\Delta s^2 < 0}$

If Δs^2 is smaller than zero in one reference frame it is smaller than zero in all other inertial reference frames. Two events for which Δs^2 is smaller than zero can always be made simultaneous in a specially selected reference frame

For two events in the "primed" reference frame, equality

$$\Delta t' = \gamma\Delta t - \gamma u \Delta x / c^2 = 0$$

holds if $u = c^2\frac{\Delta t}{\Delta x}$ Because $|u| < c$ one gets $\left|\frac{\Delta t}{\Delta x}\right| < \frac{1}{c}$

and consequently $\Delta s^2 \equiv c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 < c^2\Delta t^2 - \Delta x^2 < 0$

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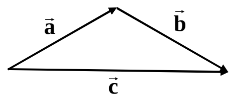
Vectors

Geometrical vectors



$$\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$$
$$\vec{\mathbf{a}} - \vec{\mathbf{a}} \equiv \vec{\mathbf{0}}$$

Given a coordinate system, each vector is identified with its 3 components: $\vec{\mathbf{a}} \to (a_1, a_2, a_3)$

A set of numbers can be identified with a vector but only if these numbers can be transformed to a different coordinate system according to vector coordinate transformation.

Such transformations are given by relations

$a'_m = \sum_{n=1}^{3} p_{mn} a_n$, where 9 numbers p_{mn} are the same for all 3-number sets.

There must be one to one correspondence between triplets (a'_1, a'_2, a'_3) and (a_1, a_2, a_3).

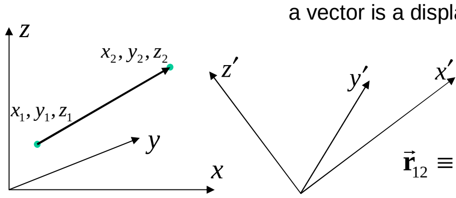
Therefore $\det(p_{mn}) \neq 0$

Vectors can be added and can be multiplied by a number

$\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} \to \vec{\mathbf{c}} \equiv (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ and $\alpha\vec{\mathbf{c}} \equiv (\alpha c_1, \alpha c_2, \alpha c_3)$

3-vectors (example)

If transformation of a coordinate system includes translation, then an example of a vector is a displacement vector



$$\vec{\mathbf{r}}_{12} \equiv (\Delta x, \Delta y, \Delta z) \equiv (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$(\Delta x, \Delta y, \Delta z)$ are transformed to $(\Delta x', \Delta y', \Delta z')$ by rotation of the coordinate system

We can get other vectors from a displacement-vector. For example,

3-velocity: $\dfrac{d\vec{\mathbf{r}}(t)}{dt} = \dfrac{d}{dt}\left(x_{12}(t), y_{12}(t), z_{12}(t)\right) \equiv (\upsilon_x, \upsilon_y, \upsilon_z) \equiv \vec{\boldsymbol{\upsilon}}$

3-momentum: $\vec{\mathbf{p}} \equiv m\vec{\mathbf{u}}$

3-acceleration: $\dfrac{d}{dt}\vec{\mathbf{u}}(t) \equiv \vec{\mathbf{a}}$

3-force: $\dfrac{d}{dt}\vec{\mathbf{p}} \equiv \vec{\mathbf{f}}$

Velocity, momentum, acceleration, and force are vectors because *t*, *m* are scalars (that is they are numbers which do not change under rotation of coordinates). Recollect that multiplication of a vector by a number (or division by a number as in the case of the derivative calculations) results in a new vector.

Vectors in Euclidian space

Under certain transformations of the coordinate system (3D rotation and translation) , the following quantity

$$|\vec{\mathbf{a}}|^2 \equiv a_x{}^2 + a_y{}^2 + a_z{}^2$$

is preserved for every ordinary 3D vector. Note that this is a not negative number.

Because

1) $|\vec{\mathbf{a}}+\vec{\mathbf{b}}|^2 = (a_x+b_x)^2 + (a_y+b_y)^2 + (a_z+b_z)^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2(a_x b_x + a_y b_y + a_z b_z)$

2) $|\vec{\mathbf{a}}|^2$, $|\vec{\mathbf{b}}|^2$ and $|\vec{\mathbf{a}}+\vec{\mathbf{b}}|^2$ do not change after rotation and/or translation,

$a_x b_x + a_y b_y + a_z b_z$ should be also invariant. This invariant is called a scalar product

$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} \equiv a_x b_x + a_y b_y + a_z b_z$$

This scalar product has the properties:

$$\vec{\mathbf{c}}\cdot\vec{\mathbf{a}} = \vec{\mathbf{a}}\cdot\vec{\mathbf{c}}$$
$$(\vec{\mathbf{a}}+\vec{\mathbf{b}})\cdot\vec{\mathbf{c}} = \vec{\mathbf{a}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}}$$
$$\vec{\mathbf{a}}\cdot(\alpha\vec{\mathbf{b}}) = \alpha\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$$

Definition of **4**-vectors

4-vectors can be added:

$$\vec{\mathbf{A}} \equiv (A_x, A_y, A_z, A_t), \vec{\mathbf{B}} \equiv (B_x, B_y, B_z, B_t)$$
$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x, A_y + B_y, A_z + B_z, A_t + B_t)$$

Coordinates of 4-vectors are transformed according Lorentz transformation

Lorentz transformations for a 4-displacement

$$\Delta\vec{\mathbf{R}} \equiv (\Delta x, \Delta y, \Delta z, c\Delta t)$$
$$\Delta x' = \gamma\left(\Delta x - \frac{u}{c} c\Delta t\right)$$
$$\Delta y' = \Delta y; \quad \Delta z' = \Delta z$$
$$c\Delta t' = \gamma\left(c\Delta t - \frac{u}{c}\Delta x\right)$$

Lorentz transformations for a general 4-vector

$$\vec{\mathbf{A}} \equiv (A_x, A_y, A_z, A_t)$$
$$A_x' = \gamma\left(A_x - \frac{u}{c} A_t\right)$$
$$A_y' = A_y; \quad A_z' = A_z$$
$$A_t' = \gamma\left(A_t - \frac{u}{c} A_x\right)$$

Note: Sometimes (when convenient) we will use for 4-vectors the notation $\vec{\mathbf{A}} \equiv (A_1, A_2, A_3, A_4)$

4-vectors in Minkovski space

We define $\vec{\mathbf{A}}\cdot\vec{\mathbf{A}} \equiv A_t^{\,2} - A_x^{\,2} - A_y^{\,2} - A_z^{\,2}$

This definition coincides with the definition of the interval and therefore does not change under Lorentz transformations. Correspondingly, a scalar product is defined by

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} \equiv A_t B_t - A_x B_x - A_y B_y - A_z B_z$$

Lorentz transformations preserve the scalar product: $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \vec{\mathbf{A}}'\cdot\vec{\mathbf{B}}'$

Other properties of scalar products are identical to the properties of ordinary scalar products in 3D space

$$\vec{\mathbf{A}}\cdot\left(\alpha\vec{\mathbf{B}}\right) = \left(\alpha\vec{\mathbf{A}}\right)\cdot\vec{\mathbf{B}} = \alpha\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)$$

$$\vec{\mathbf{A}}\cdot\left(\vec{\mathbf{B}}+\vec{\mathbf{C}}\right) = \vec{\mathbf{A}}\cdot\vec{\mathbf{B}} + \vec{\mathbf{A}}\cdot\vec{\mathbf{C}}$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$$

4-velocity

Definition of 4-velocity

$$\vec{\mathbf{V}} \equiv \frac{d}{d\tau}\left(x, y, z, ct\right) = \frac{d}{dt}\left(x, y, z, ct\right)\frac{dt}{d\tau} = \gamma(\upsilon)\cdot\left(\upsilon_x, \upsilon_y, \upsilon_z, c\right)$$

where $\gamma(\upsilon) = \left(1-\upsilon^2/c^2\right)^{-1/2}$

Square of 4-velocity equals c^2.

$$\vec{\mathbf{V}}^2 \equiv V_4^2 - V_1^2 - V_2^2 - V_3^2 = \frac{c^2 - \upsilon_x^2 - \upsilon_y^2 - \upsilon_z^2}{1-\upsilon^2/c^2} = c^2$$

This also can be seen almost immediately if we do calculations in a reference frame where the **3**-velocity is zero.

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