SPECIAL RELATIVITY
IMPORTANT RELATIONS AND DIFINITIONS

All transformations are written for the standard configuration of the primed and not primed reference frames: the primed reference frame is moving in x-direction with velocity u.

LORENTZ TRANSFORMATIONS : the origins of the two reference frames coincide at $t = t' = 0$

$$\begin{cases} x' = \gamma\left(x - ut\right) \\ y' = y; \quad z' = z \qquad \gamma = \dfrac{1}{\sqrt{1-u^2/c^2}} \\ t' = \gamma\left(t - \dfrac{u}{c^2}x\right) \end{cases}$$

4-VECTOR TRANSFORMATIONS:

$$\begin{cases} A'_x = \gamma\left(A_x - \dfrac{u}{c}A_t\right) \\ A'_y = A_y; \quad A'_z = A_z \quad \gamma = \dfrac{1}{\sqrt{1-u^2/c^2}} \\ A'_t = \gamma\left(A_t - \dfrac{u}{c}A_x\right) \end{cases}$$

EXAMPLES OF 4-VECTORS:

$$\mathbf{\Delta}\vec{\mathbf{R}} \equiv \left[\Delta x, \Delta y, \Delta z, c\Delta t\right];$$

$$\vec{\mathbf{V}} \equiv \frac{d}{d\tau}\left[\Delta x, \Delta y, \Delta z, c\Delta t\right] = \gamma(\upsilon)\cdot\left[\upsilon_x, \upsilon_y, \upsilon_z, c\right]; \quad \gamma(\upsilon) = \frac{1}{\sqrt{1-\upsilon^2/c^2}}$$

$$\vec{\mathbf{K}} \equiv \left[\mathbf{k}, \frac{\omega}{c}\right];$$

$$\vec{\mathbf{P}} \equiv m\vec{\mathbf{V}} \equiv \left(\vec{\mathbf{p}}, E/c\right);$$

$$\vec{\mathbf{A}} = \frac{d\vec{\mathbf{V}}}{d\tau} = \gamma\frac{d\gamma}{dt}\left[\vec{\boldsymbol{\upsilon}}, c\right] + \gamma^2\left[\vec{\mathbf{a}}, 0\right]; \quad \gamma = \frac{1}{\sqrt{1-\upsilon^2/c^2}}; \quad \frac{d\gamma}{dt} = \frac{\upsilon/c^2}{\left(1-\upsilon^2/c^2\right)^{3/2}}\frac{d\upsilon}{dt};$$

$$\vec{\mathbf{F}} \equiv \frac{d}{d\tau}\vec{\mathbf{P}} = \gamma\left(\upsilon\right)\left[\vec{\mathbf{f}}, \frac{1}{c}\frac{dE}{dt}\right] = \frac{dm}{d\tau}\vec{\mathbf{V}} + m\vec{\mathbf{A}}; \qquad \gamma(\upsilon) = \frac{1}{\sqrt{1-\upsilon^2/c^2}};$$

RELATIVISTIC TRANSFORMATIONS OF 3-VECTORS

VELOCITY TRANSFORMATIONS:

$$\begin{cases} \upsilon'_1 = \dfrac{\upsilon_1 - u}{1 - u\upsilon_1/c^2} \\ \upsilon'_2 = \dfrac{1}{1 - u\upsilon_1/c^2}\cdot\dfrac{\upsilon_2}{\gamma} \qquad \gamma = \dfrac{1}{\sqrt{1-u^2/c^2}} \\ \upsilon'_3 = \dfrac{1}{1 - u\upsilon_1/c^2}\cdot\dfrac{\upsilon_3}{\gamma} \end{cases}$$

WAVE VECTOR AND FREQUENCY TRANSFORMATIONS (PLANE WAVE):

$$\begin{cases} k_x' = \gamma\left(k_x - \dfrac{u}{c^2}\omega\right) \\ k_y' = k_y \\ k_z' = k_z \end{cases} \qquad \gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

$$\omega' = \gamma\left(1 + \frac{u}{\upsilon}\cos\theta\right)\omega$$

$$\tan\theta' = \frac{k_y'}{k_x'} = \frac{\sin\theta}{\gamma\left(\cos\theta + \dfrac{u\upsilon}{c^2}\right)}$$

$$\upsilon' = \frac{\gamma\left(\upsilon + u\cos\theta\right)}{\sqrt{\dfrac{\gamma^2}{c^2}\left(\upsilon + u\cos\theta\right)^2 + 1 - \dfrac{\upsilon^2}{c^2}}}$$

FORCE AND POWER TRANSFORMATIONS :

$$\begin{cases} f_1' = \dfrac{1}{1-\upsilon_1 u/c^2}\left(f_1 - \dfrac{u}{c^2}\dfrac{dE}{dt}\right) \\ f_2' = \dfrac{1}{\gamma\left(1-\upsilon_1 u/c^2\right)} f_2; \\ f_3' = \dfrac{1}{\gamma\left(1-\upsilon_1 u/c^2\right)} f_3 \\ \dfrac{dE'}{dt'} = \dfrac{1}{1-\upsilon_1 u/c^2}\left(\dfrac{dE}{dt} - u f_1\right) \end{cases} \qquad \gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

$$\text{If } \frac{dm}{d\tau} = 0 \quad \text{then} \quad \frac{dE}{dt} = \vec{\mathbf{f}} \cdot \vec{\mathbf{\upsilon}}$$

EM FIELD TRANSFORMATIONS:

$$e_x' = e_x$$

$$e_y' = \gamma\left(e_y - u b_z\right) \qquad \gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

$$e_z' = \gamma\left(e_z + u b_y\right)$$

$$b_x' = b_x$$

$$b_y' = \gamma\left(b_y + e_z u/c^2\right)$$

$$b_z' = \gamma\left(b_z - e_y u/c^2\right)$$

LORENTZ FORCE

$$\vec{\mathbf{f}} = q\vec{\mathbf{\upsilon}} \times \vec{\mathbf{b}} + q\vec{\mathbf{e}}$$