

## Chapter 4: Damping

### 4.1: The Absorbing Atom and its Environment

In chapter 3, we saw how the absorption coefficient for a non-interacting atomic species at rest is given by the combination of the strength of the transition, the population of valid absorbers, a correction for stimulated emission and a line profile function describing the frequency dependence of the transition. The line profile function for an isolated atom at rest is given by equation (3-60):

$$\phi(\lambda) = \frac{\frac{\Delta\lambda}{2\pi}}{(\lambda - \lambda_0)^2 + \left(\frac{\Delta\lambda}{2}\right)^2}. \quad (4-1)$$

Perturbing atoms will not affect the basic physical processes that give rise to this profile, but they can alter the physical parameters that control it, namely  $\Delta\lambda$  (altering the width of the line) and  $\lambda_0$  (shifting the wavelength of the line). Thus the line profile function for a single atom can be altered by interactions with its environment.

As we do not have a single absorber, but an ensemble of absorbers with each absorber in different conditions, each absorbing atom can have a different line profile function. The effect seen will be a convolution of the line profile function as affected by the environment and the probability distribution function of the environment:

$$\begin{aligned} \phi(\lambda) &= \phi_{\text{isolated}}(\lambda) \otimes W(\text{environment}) \\ &= \int_{\substack{\text{all possible} \\ \text{conditions}}} \phi_{\text{isolated}}(\text{wavelength shifted to } \lambda \text{ by environment}) W(\text{environment}) d\text{environment} \end{aligned} \quad (4-2)$$

#### **4.2: The Lorentzian Line Width**

The width of the Lorentz profile of the line will be altered by any process which changes the lifetimes of either the upper or lower energy states of the transition. The natural width of the line is given by equation (3-61) as

$$\Delta\lambda = \frac{\lambda^2\Gamma}{2\pi c} \quad (4-3)$$

where the damping constant  $\Gamma$  is the sum of the rates at which atoms in the upper and lower levels change state (thus being equal to the sum of the reciprocals of the lifetimes for each level). The only way in which a non-interacting atom can change state, or otherwise affect the lifetime of the transition, is by spontaneous emission. For such an isolated atom,  $\Gamma$  is the sum of the spontaneous emission rates for each level.

Although  $\Gamma$  is dependent solely on the level lifetimes for an isolated atom,  $\Gamma$  is a property of the transition between the two levels, rather than a property of the levels. Thus,  $\Gamma$  can be considered to depend on the lifetime of the transition. This is an important distinction when interactions with other particles (i.e. collisions) are considered. Only a small fraction of collisions will depopulate the upper or lower level (non-adiabatic collisions), most collisions will be adiabatic. An adiabatic collision will, in general, alter the transition energy (and thus the wavelength) for the duration of the collision. If this shift in energy is great enough, it can be considered as an interruption in the existence of the transition, and will thus affect the lifetime of the transition, and therefore,  $\Gamma$ . This will be considered in more detail in section 4.5.

The transition lifetime of a non-isolated atom can be affected in a number of ways: spontaneous emission, interaction with the radiation field (absorption or stimulated emission) and collisions. The damping constant for the transition will be given by the sum of the rates for these processes:

$$\Gamma = \Gamma_R + \Gamma_A + \Gamma_C \quad (4-4)$$

where  $\Gamma_R$  is the total spontaneous emission rate,  $\Gamma_A$  is the combined absorption and stimulated emission rate<sup>1</sup> and  $\Gamma_C$  is the rate of occurrence of significant collisions.

#### 4.2.1: Spontaneous Emission

All downward transitions from both the upper and lower energy states need to be considered. The sum of all of the possible spontaneous transition rates from a level is simply given by the natural lifetime of the level:

$$\Gamma_R = \frac{1}{t_N} \quad (4-5)$$

where  $t_N$  is the lifetime. The total spontaneous emission rate taking both levels into account will be given by a combination of the two level lifetimes involved:

$$\Gamma_R = \Gamma_R^{\text{Upper level}} + \Gamma_R^{\text{Lower level}} \quad (4-6)$$

In the solar photosphere, spontaneous emission will not strongly affect the line width, as the effect of collisions will be much greater.<sup>2</sup>

#### 4.2.2: Absorption and Stimulated Emission

When there is a radiation field present, atomic level lifetimes will be affected by stimulated emission and absorption. The rates of these processes will be related to the intensity of the radiation field at the transition wavelength. The stimulated absorption rate will be given by

$$\Gamma_{ij} = \int_{\text{all directions}} B_{ij} I_\lambda d\omega \quad (4-7)$$

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<sup>1</sup>The radiative contributions to the damping constant  $\Gamma$ , namely  $\Gamma_R$  (with “R” for radiation) and  $\Gamma_A$  are small in the solar photosphere. The damping width of the line will be determined by collisions.

<sup>2</sup>This will not be the case for all conditions, such as in nebulae, where collision rates are much lower and spontaneous emission rates become correspondingly more important. If the spontaneous emission and radiative rates are not small compared to collisional rates, LTE does not occur. The existence of LTE thus guarantees small radiative rates.

where  $d\omega$  is the solid angle into which the specific intensity  $I_\lambda$  passes and the integral is carried out over all directions. The stimulated emission rate is

$$\Gamma_{ji} = \oint_{\text{all directions}} B_{ji} I_\lambda d\omega. \quad (4-8)$$

To estimate the contribution of stimulated emission and absorption to the level lifetime, we can assume that  $I_\lambda$  is isotropic and approximately equal to  $B_\lambda$ . The absorption and emission rates are then given by

$$\begin{aligned} \Gamma_{ij} &= 4\pi B_{ij} B_\lambda \\ &= \frac{8\pi hc^2}{\lambda^5} \frac{B_{ij}}{e^{hc/\lambda kT} - 1} \end{aligned} \quad (4-9)$$

for absorption, and

$$\Gamma_{ji} = \frac{8\pi hc^2}{\lambda^5} \frac{B_{ji}}{e^{hc/\lambda kT} - 1} \quad (4-10)$$

for emission.

In LTE, the relationship between the spontaneous emission rates and the rates above can be obtained from equation (3-33), which gives

$$A_{ji} = (e^{hc/\lambda kT} - 1) B_{ji} B_\lambda. \quad (4-11)$$

Considering the ratio of the spontaneous emission rate for this transition to the stimulated emission rate, we obtain

$$\frac{A_{ji}}{\Gamma_{ji}} = \frac{1}{4\pi} (e^{hc/\lambda kT} - 1). \quad (4-12)$$

For visible wavelengths and photospheric temperatures, the spontaneous emission rate is many times greater than the stimulated emission rate.

Transitions (spontaneous and stimulated) between the upper level and other levels, and between the lower level and other levels will also affect the level lifetimes. A similar relationship between spontaneous and stimulated emission and absorption rates will exist for these transitions. Thus, in the solar photosphere, the contribution of stimulated emission and absorption to the width of the line is much less than the contribution due to spontaneous emission (the natural lifetime). As the natural line width (due solely to spontaneous emission) is small compared to the line width in the photosphere (due mainly to collisions), the effects of stimulated emission and absorption on the line width can be safely neglected.

### 4.2.3: Collisions

Interactions between the absorbing atom and the surrounding particles dominate the width of the Lorentz profile of the line. Of the mechanisms contributing to the width of the line, as well as being the most important, it is also the least well described theoretically, as atoms generally are sufficiently complex so as to defy simple quantum mechanics. In general, each type of perturber must be taken into account, along with any interactions between the perturbers. However, suitable approximations can be made so as to make the problem more approachable.

### 4.2.4: Transition Lifetime and Asymmetry

It is usually assumed that the collision rates will be independent of the wavelength across the small wavelength range of the transition. This gives a wavelength independent (as far as the particular line is concerned) value of  $\Delta\lambda$ . Thus, although the width of the line will change if the lifetime changes, the Lorentzian profile will remain completely symmetric. Although determination of collision rates is in general important to the theory of spectral line formation, as they will not cause any asymmetry of the line, their accuracy is not so important here as any discrepancy between theory and observation can be compensated for by adjusting the value of  $\Gamma$  used so as to reproduce the observations. For other purposes, accurate knowledge of collision rates is important, such as when accurate abundances are being determined, as the abundance determined from a spectral line depends on the damping rates, particularly for strong lines.

### 4.3: The Transition Wavelength

The wavelength of the transition,  $\lambda_0$ , will be affected by any external electric field; this change in wavelength is called the **Stark effect**.<sup>3</sup> In hydrogen and helium, spectral lines are observed to split into a number of components, with the splitting proportional to the field strength; this is called the first order, or **linear Stark effect**. With other elements, the splitting is negligible and what is observed is a shift of the wavelength of the line, usually towards longer wavelengths. This shift is proportional to the square of the magnitude of the electric field and is called the **quadratic Stark effect**.

#### 4.3.1: The Stark Effect<sup>4</sup>

The energy  $E_i$  of an atom in state  $i$  in an external electric field  $E$  can be written as

$$E_i = E_{0i} + A_i E + B_i E^2 + C_i E^3 + \dots \quad (4-13)$$

where  $E_{0i}$  is the unperturbed energy of the atom and  $A_i$ ,  $B_i$  etc. are constants dependent on the state of the atom. When the first perturbation term is dominant, a first order Stark effect will be seen, and if the second is dominant, a quadratic Stark effect will be observed. The first order Stark effect predominates when levels of opposite parity are nearly degenerate. These levels are then shifted in opposite directions by the field.

When such levels are further apart ( $\sim 100 \text{ cm}^{-1}$ ), they do not interact, and respond differently to the external electric field, and no first order Stark effect is seen. This is the usual case for atoms other than hydrogen and helium which show a quadratic Stark effect.

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<sup>3</sup>Named after Stark who first observed it in 1913.

<sup>4</sup>The Stark effect will only be briefly discussed here. Many works on the quantum theory of atoms only briefly discuss the Stark effect, or ignore it altogether. A more complete, but still simple, treatment can be found in White, H.E. "Introduction to Atomic Spectra", McGraw-Hill (1934).

As the energy shifts of the upper and lower levels of a transition will differ, the transition wavelength will be altered.

#### **4.3.2: The Line Profile and the Stark Effect**

The shift in wavelength of a single atom can be found from the local electric field, and the shift of the entire line can be found in terms of an average magnitude of the electric field. The shape of the line profile will also be affected, as each individual absorbing atom will have its line profile shifted by a various due to differing local electric fields. The resultant line profile of the ensemble of absorbers will be a combination of these individual shifted profiles.

#### **4.3.3: Asymmetry and Wavelength Shift**

As there will be electric fields present in the photosphere due to both charged particles (electrons and ions) and dipoles (neutral atoms, especially hydrogen), there will be changes in the wavelengths of lines. The electric fields are due to microscopic fluctuations in the distribution of particles, and the distribution of the electric field will be very asymmetric, and any effect will contribute to the asymmetry in spectral lines. The effects would be expected to be quite small in overall effect, but should be taken into account as a possible source of asymmetry.

#### **4.4: Damping Theory**

Formulating a complete theory of damping is a formidable problem, and is not even usually attempted. Numerous approximations are usually introduced to make the problem more tractable. Some of the approximations traditionally made are sound, but others excessively reduce the accuracy of the resulting theory, or limit its applicability to special cases only.

Damping theory is usually approached from one of two viewpoints: the **impact approximation** wherein individual encounters with perturbers are considered, and the **quasi-static approximation** where all perturbers are considered simultaneously but the motion of the perturbers is neglected. Each of these approaches has its own strengths and drawbacks.

#### **4.4.1: The Impact Approximation**

In the impact approximation, only the effects of close encounters with perturbing particles are considered. Impact broadening theory thus consists of determining the rate of significant encounters, and the cumulative effect of such significant encounters. Each encounter is assumed to be with a single perturbing particle, and the time taken for an encounter is assumed to be small compared to the times between encounters (which is necessary for encounters to be with single perturbers).

A number of conditions must be satisfied for the impact approximation to be valid. The perturber density must be low enough so that the average perturber distance is large enough for the effect of distant perturbers to be small, the effect of a close encounter with a perturber must be greater than the effects of distant perturbers, and the close encounter must take place sufficiently rapidly. The first condition (perturber density) is most readily satisfied by the less abundant perturbers (any other than neutral hydrogen), the second by perturbers with predominantly short-range effect (such as neutral atoms), and the third (high speed) by particles of low mass (particularly electrons). We can thus expect broadening by electrons to be well described by the impact approximation, while that due to positive ions to be less so. Particle densities in the photosphere are low, and temperatures are high, so we can expect broadening by neutral atoms, particularly light atoms, to be treatable in the impact regime.

The simplest impact broadening theories assume the effects of all encounters are identical, reducing the problem to finding the collision rate. More sophisticated theories account for differences between individual encounters. The problem of the

interaction between the perturber and the absorber is quite complex; a successful modern theory (the Brueckner-O'Mara theory)<sup>5</sup> is discussed in section 4.5, along with simpler theories.

Most theories assume that the perturbing particle moves in a straight line and its motion is unaffected by the perturbed absorber (the **straight classical path approximation**). The effects of removing this approximation are quite small as the perturber motion must be close to a straight path when the impact approximation regime is valid.

#### 4.4.2: The Quasi-Static Approximation

The impact approximation ignores the effects of distant perturbers. These perturbers do, however, have an effect, which can be important if the effects of the closest distant perturbers are sufficiently large. This is most likely for abundant perturbers, such as neutral hydrogen, or long range perturbers such as ions and electrons. Quasi-static damping theory, or **statistical broadening theory**, is an attempt to account for simultaneous effects of multiple perturbers. As such, apart from determining the effect of a single perturber, the positions of all of the perturbers must be taken into account.

Although the probability of different perturber configurations can be readily determined, it would be a much more difficult problem to deal with the evolution of the perturber positions over time, so it is assumed that the motion of the perturbers is small during the time in which the perturbation is important.

As many perturbers are considered at once, and emphasis is on the effect of distant perturbers, the interaction between a single perturber and the absorbing atom is usually considered in simple terms. For close encounters between perturber and absorber, we can expect a simple picture of the interaction to be invalid, but the quasi-static approximation itself ceases to be valid in such cases.

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<sup>5</sup>See, for example, Anstee, S.D. and O'Mara, B.J. "An Investigation of Brueckner's Theory of Line Broadening with Application to the Sodium *D* Lines" *Monthly Notices of the Royal Astronomical Society* **253**, pg 549-560 (1991).

A problem with standard statistical broadening theory is that it is incorrect. Most work on quasi-static broadening has been for the broadening of spectral lines (particularly first order Stark broadening of hydrogen)<sup>6</sup> by highly ionised dense plasmas, and while it is generally correct for such cases, it is not correct for quadratic Stark broadening of heavier elements in the photosphere. A more appropriate theory is developed in section 4.6.

#### 4.4.3: Combining the Impact and Quasi-Static Approximations

The impact approximation fails to take multiple simultaneous interactions into account; such interactions will dominate while the nearest perturbing particle is a long way from the absorber. The quasi-static approximation completely ignores the time distribution of events and fails for close encounters with perturbers. A simple first-order combination of the two approaches can be made by using impact theory to deal with close encounters and using statistical broadening theory to account for the effects of distant perturbers between close encounters.

As the quasi-static theory fails to predict any effect on the level lifetime, impact theory must be used to determine  $\Gamma$  and the Lorentzian line width  $\Delta\lambda$ . If quasi-static theory is used to find the line shift, the resultant line profile (as found using equation 4-2) becomes the distribution of line shifts  $W(\lambda_0)$  as given by the quasi-static theory convoluted with the Lorentzian profile given by the impact theory:

$$\begin{aligned}\phi(\lambda) &= \int_0^{\infty} \phi_L(\lambda, \Delta\lambda, \lambda_0) W(\lambda_0) d\lambda_0 \\ &= \int_0^{\infty} \frac{\frac{\Delta\lambda}{2\pi}}{(\lambda - \lambda_0)^2 + \left(\frac{\Delta\lambda}{2}\right)^2} W(\lambda_0) d\lambda_0.\end{aligned}\tag{4-14}$$

In section 4-6, it will be shown that the line shift distribution  $W(\lambda_0)$  is asymmetric, and thus should be taken into account in any study of asymmetry, but is difficult to

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<sup>6</sup>See Griem, H.R. "Spectral Line Broadening by Plasmas" Academic Press, New York (1974).

calculate. The possible contribution to the total asymmetry of the line due to damping is then examined in section 4-7.

In the photosphere, we can expect  $W(\lambda_0)$  to be a narrow function, centred on a wavelength very close to the unperturbed  $\lambda_0$ . Thus, the damped line profile will be close to Lorentzian, and for most purposes, it will be sufficient to assume that it is exactly Lorentzian. The non-Lorentzian contribution under other conditions can be much larger, but, in such cases, both the impact and quasi-static approximations will tend to fail (due to simultaneous multiple close encounters with fast moving perturbers) so this simple combination of the two theories will also not be valid.

#### 4.4.4: Damping by Various Types of Perturbers

The interactions due to different types of perturbers are usually treated separately. The two main types of collisional damping, namely damping due to interactions with charged particles (ions and electrons) and damping due to neutral atoms (especially hydrogen). Damping caused by charged particles is called Stark broadening, while damping by neutral hydrogen is termed **van der Waals broadening**. Despite this nomenclature, both forms of damping have their origins in the Stark effect.<sup>7</sup> Any form of damping where the perturber affects the absorber via electric fields can be considered to be effected by the Stark effect. As the fundamental mechanism giving rise to the damping in these two cases (which involve Coulomb fields and dipole fields respectively) is the same, it should be possible to treat them simultaneously with a unified theory. This is explored later in this chapter.

With the quadratic Stark effect, the shift due to an external electric field of magnitude  $E$  can be given in terms of a constant for the level, so for a transition from a level  $i$  to a level  $j$ , the energy shifts are given by

$$\begin{aligned}\Delta E_i &= C_i E^2 \\ \Delta E_j &= C_j E^2.\end{aligned}\tag{4-15}$$

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<sup>7</sup>White, for example, makes this point in White, H.E. "Introduction to Atomic Spectra" McGraw-Hill (1934) (see pages 431 and 436).

The constant is normally given in units of  $\text{cm}^{-1}$  per  $100 \text{ kVcm}^{-1}$ . The transition energy is then changed by the difference between the shifts of each level, so

$$\begin{aligned}\Delta E_{ij} &= E_j - E_i \\ &= (C_j - C_i)E^2 \\ &= C_{ij}E^2\end{aligned}\quad (4-16)$$

and as the shift is small compared to the total energy, the wavelength shift is given by

$$\Delta\lambda = C_s E^2 \quad (4-17)$$

where the Stark constant for the wavelength shift is related to the constant for the energy shift by

$$C_s = \frac{\lambda^2 C_{ij}}{hc} \quad (4-18)$$

The shift of the upper level will generally be greater than that of the lower level.<sup>8</sup>

For electric fields due to charged particles, the overall effect will vary as the inverse fourth power of the separation

$$\Delta\lambda = \frac{C_4}{r^4} \quad (4-19)$$

and for neutral atoms, with predominantly dipole electric fields, the wavelength shift will be

$$\Delta\lambda = \frac{C_6}{r^6} \quad (4-20)$$

These are the usual equations for Stark broadening by charged particles and for van der Waals broadening.

While the Stark effect is well understood for uniform electric fields, the fields due to very close encounters will be far from uniform. Further difficulties arise as at short ranges, the perturber and absorber will strongly affect each other, and the electric field produced by the perturber will be affected by the absorbing atom. Short range encounters (for which impact broadening theory is used) will therefore be more complicated, and great care must be taken in describing the interaction in terms of a single constant. For quasi-static broadening, where long range interactions are dealt with, a simple treatment of the perturbing particle should suffice. In the photosphere,

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<sup>8</sup>See, for example, Babcock, H.D. "The Effect of Pressure on the Spectrum of the Iron Arc" *The Astrophysical Journal* **67**, pg 240-261 (1928).

as the major neutral atomic species is hydrogen, the simple dipole description (equation (4-20) ) will be adequate. This will not necessarily be the case for other perturbers, such as helium, where non-dipole contributions to the instantaneous electric field are important.

#### 4.4.5: Broadening of Hydrogen Lines

As hydrogen energy levels will be split into a number of components by the linear Stark effect rather than shifted by the quadratic Stark effect, the procedure for the computation of broadening of hydrogen lines is somewhat different to that for spectral lines of other elements. The resultant line profile can be found as a combination of the separate Stark components into which the line is split, each with its own wavelength shift distribution. Each of the Stark components can be found in a manner similar to other lines, with the instantaneous shift being proportional to the perturbing electric field rather than its square.<sup>9</sup>

As no hydrogen line profiles are examined in this work, the broadening of hydrogen lines is not of great interest here.

#### 4.5: Impact Broadening Theory

The usual impact approximation broadening theory considers the atom to be a classical oscillator at a particular frequency. A perturbing particle will cause a shift in this frequency of

$$\Delta\omega = \frac{C_p}{r^p} \quad (4-21)$$

where  $r$  is the distance to the perturber and  $C$  and  $p$  are constants depending on the type of interaction involved (see table 4-1). If the perturbing particles travel in straight lines (the straight classical path approximation), with a closest approach to the

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<sup>9</sup>For a full treatment of the broadening of hydrogen lines, see Griem, H.R. "Spectral Line Broadening by Plasmas" Academic Press, New York (1974).

oscillator of  $\rho$ , the passage of such a perturber will cause a phase shift  $\eta(t)$  in the oscillation, dependent on the total time taken by the encounter and the strength of the instantaneous frequency shift.

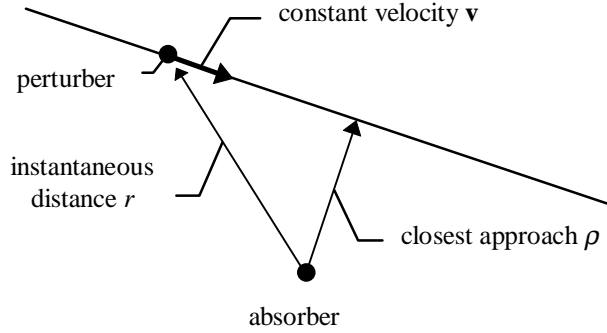


Figure 4-1: The Straight Classical Path

The total phase shift induced by a perturber at time  $t$  is

$$\begin{aligned}\Delta\eta(t) &= \int_{-\infty}^t \Delta\omega(t') dt' \\ &= C_p \int_{-\infty}^t (\rho^2 + v^2 t'^2)^{-p/2} dt'\end{aligned}\quad (4-22)$$

which can be readily evaluated to give

$$\eta(t) = \frac{C_p \psi_p}{v \rho^{p-1}} \quad (4-23)$$

where  $\psi_p$  is a constant dependent on  $p$  (see table 4-1).

Table 4-1:  $p$  and the Type of Interaction.

$p$	Absorber	Perturber	$\psi_p$
2	Hydrogen	Charged particle	$\pi$
3	Neutral atom	Same atom	2
4	Atom other than hydrogen	Charged particle	$\pi/2$
6	Atom other than hydrogen	Neutral atom	$3\pi/8$

One approach from here is to assume that only encounters causing a phase shift greater than some phase shift  $\eta_o$  will contribute to the broadening of the line. In the **Weisskopf approximation**, it is assumed that  $\eta_o = 1$  and that the phase change is instant, breaking the oscillation into discrete segments (see figure 4-2).

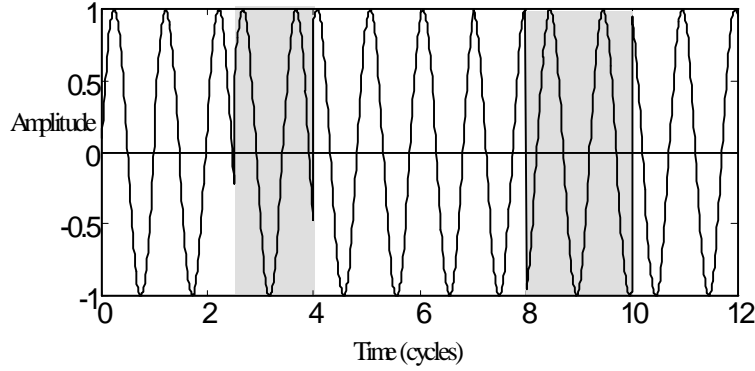


Figure 4-2: Effect of Instantaneous Impacts on Oscillation

The effect of this on the frequency of the oscillator is the same as the effect of the finite lifetime of the level; the lifetime of the transition is reduced by the collisions causing significant changes in phase. The rate of significant collisions is

$$\Gamma_c = 2\pi N \rho_o^2 \bar{v} \quad (4-24)$$

where  $\rho_o$  is the closest approach needed to produce the minimum effective phase shift.

$$\rho_o = \left( \frac{C_p \Psi_p}{\eta_o \bar{v}} \right)^{\frac{1}{p-1}}, \quad (4-25)$$

and the mean relative speed of the perturber is given by

$$\bar{v} = \sqrt{\frac{8kT}{\pi} \left( \frac{1}{M_{\text{absorber}}} + \frac{1}{M_{\text{perturber}}} \right)}. \quad (4-26)$$

For the impact approximation to be valid, the collisions must be rapid and not overlap in time with each other. Such separation of the collisions in time requires  $\rho_o$  to be much less than the mean interparticle distance.

This simple theory gives results that are only of the right order of magnitude, even if the damping constant  $C_p$  is known, and as the cutoff phase shift for a collision to be effective is arbitrary, and is assumed to be sharp, with no contribution at all from

the (numerous) collisions with lesser phase shifts, the results can scarcely be expected to be more accurate.

#### 4.5.1: Lindholm-Foley Theory

The frequency of an oscillator at any time is given by the unperturbed frequency  $\omega_0$  and the frequency shift  $\Delta\omega$  given by equation (4-21), so

$$\omega(t) = \omega_0 + \Delta\omega(t). \quad (4-27)$$

The instantaneous phase of the oscillator is given by

$$\begin{aligned} \eta(t) &= \eta(t=0) + \int_0^t \frac{d\eta(t')}{dt'} dt' \\ &= \eta(0) + \int_0^t (\omega_0 + \Delta\omega(t')) dt' \\ &= \eta(0) + \omega_0 t + \Delta\eta(t) \end{aligned} \quad (4-28)$$

where  $\Delta\eta$  is contribution to the phase due to collisions, the contribution for a single collision being given by equation (4-23). The line profile is given by the Fourier transform of the oscillation, which is given by

$$\phi(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \left| \int_{-T/2}^{T/2} e^{i\eta(t)} e^{-i\omega t} dt \right|^2. \quad (4-29)$$

As the collisions can be considered to occur separately in time, we can consider the sum of Fourier transforms of individual collisions, so equation (4-29) becomes

$$\begin{aligned} \phi(\omega) &= \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \left| \int_{-T/2}^{T/2} \left( \lim_{N \rightarrow \infty} \sum_{n=1}^N e^{i\eta_n(t)} \right) e^{-i\omega t} dt \right|^2 \\ &= \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \left| \int_{-T/2}^{T/2} \langle e^{i\eta_n(t)} \rangle e^{-i\omega t} dt \right|^2. \end{aligned} \quad (4-30)$$

As the encounters causing phase shifts (i.e. the times when  $\Delta\omega > 0$ ) are randomly distributed in time and the instantaneous phase is uncorrelated with the phase change caused by a particular perturber, the average oscillation in equation (4-30) can be replaced by an oscillation including an average phase factor (this is the Lindholm-Foley approximation). Then, we have

$$\phi(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \left| \int_{-T/2}^{T/2} e^{i\omega t} \langle e^{i\Delta\eta} \rangle e^{-i\omega t} dt \right|^2. \quad (4-31)$$

It would prove difficult to calculate the time average of the phase, so it is replaced by a frequency average (via the ergodic hypothesis). In this case, the frequency average is actually an average over impact parameters, which, as the rate of occurrence of impacts at an impact parameter  $\rho$  is  $2\pi\rho d\rho N\bar{v}$ , is

$$\langle e^{i\Delta\eta} \rangle_{\rho} = 2\pi N\bar{v} \int_0^{\infty} e^{i\Delta\eta(\rho)} \rho d\rho. \quad (4-32)$$

This can then be substituted into equation (4-31). The effects of the damping can be readily seen by comparing the resultant expression to equation (3-51) which was used to find the line profile function for an isolated stationary atom. The imaginary component of the complex term gives the frequency of a Lorentzian profile, and the real part gives the Lorentzian width.

This gives a damping constant of

$$\Gamma_c = 8\pi N\bar{v} \int_0^{\infty} \sin^2 \left( \frac{\eta(\rho)}{2} \right) \rho d\rho \quad (4-33)$$

and a line shift of

$$\Delta\omega_0 = 2\pi N\bar{v} \int_0^{\infty} \sin \eta(\rho) \rho d\rho. \quad (4-34)$$

These give, for the case of atoms (other than hydrogen) interacting with charged particles (each type of which needs to be considered separately as they will have differing mean speeds),

$$\Gamma_c = 11.36 C_4^{2/3} \bar{v}^{-1/3} N \quad (4-35)$$

and

$$\Delta\omega_0 = 9.85 C_4^{2/3} \bar{v}^{-1/3} N. \quad (4-36)$$

For the  $p = 6$  case, (where hydrogen will be the dominant perturber due to its abundance) we obtain

$$\Gamma_c = 8.08 C_6^{2/5} \bar{v}^{-3/5} N \quad (4-37)$$

and

$$\Delta\omega_0 = 2.94 C_6^{2/5} \bar{v}^{-3/5} N. \quad (4-38)$$

The problems with this approach are that it still does not consider overlapping collisions, which will occur especially for encounters at large distances causing small

perturbations, and still assumes that all particles (of a particular type) can be adequately described as moving at a uniform speed. The frequency at which strong collisions occur, which determines  $\Gamma$ , should be given reasonably accurately, but as the times between strong collisions, when a large number of distant perturbers have a cumulative effect, are not considered, the line shift may not be given accurately by this theory. The broadening, being purely Lorentzian (with a wavelength shift), is symmetric.

#### 4.5.2: Damping Constants for Collisions with Neutral Hydrogen

Determining damping constants accurately is a difficult task. It is made somewhat easier in the case of the photosphere by the fact that almost all neutral atomic perturbers (namely hydrogen and helium) will be in the ground state due to the high energy of the first excited state above the ground state. This simplifies the quantum mechanical problem of the absorber-perturber interaction considerably. The difficulty lies in adequately describing the absorbing atom.

As a crude approximation, we can assume that the absorber in state  $i$  can be described by an effective principal quantum number  $n_i^*$ , given by

$$n_i^* = Z \sqrt{\frac{\chi_H}{\chi_1 - \chi_i}} \quad (4-39)$$

where  $Z$  is the effective nuclear charge,  $\chi_H$  is the ionisation energy for hydrogen,  $\chi_1$  is the ionisation energy for the absorbing atom, and  $\chi_i$  is the energy of the state  $i$ .

This **hydrogenic approximation** can then be used to determine the energy shift of the state  $i$  due to the perturbation.<sup>10</sup> The energy shift is

$$\Delta E = -\frac{e\alpha\alpha_0^2 R_i^2}{r^6} \quad (4-40)$$

where  $\alpha$  is the polarisability of hydrogen ( $= 6.70 \times 10^{-25} \text{ cm}^3$ ) and

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<sup>10</sup>The hydrogenic approximation is commonly used because it is particularly simple. For a full derivation, see, for example, pg 297 in Mihalas, D. "Stellar Atmospheres" Freeman (1970). Unfortunately, the hydrogenic approximation is not always accurate.

$$\overline{R_{i(n^*,l)}^2} = \frac{n^{*2}}{2Z^2} (5n^{*2} + 1 - 3l(l+1)). \quad (4-41)$$

This gives the damping constant

$$C_6 = 4.05 \times 10^{-33} (\overline{R_{upper}^2} - \overline{R_{lower}^2}). \quad (4-42)$$

While damping constants derived in this manner can be used as approximate values, they are generally insufficiently accurate when the damping must be well-known. For more accurate damping constants, more sophisticated theory, such as that of Brueckner, as extended by O'Mara<sup>11</sup> must be used.

### 4.5.3: Brueckner-O'Mara Theory

The Brueckner-O'Mara theory has so far proved to be an accurate and reliable method for determining damping constants, particularly for collisions with neutral atomic hydrogen in the ground state.

The theory assumes that the interaction between the absorber and the hydrogen atom is sufficiently weak so that perturbation theory can be used. Rayleigh-Schrödinger perturbation theory is used and exchange interactions are neglected. The Unsöld approximation<sup>12</sup> is used in second-order perturbation theory to replace the energy denominator with a suitable average energy (which allows major simplification). Lastly, the classical path approximation is assumed to be valid.

The Hamiltonian for the absorber-perturber system can be written as

$$H = H_0 + V \quad (4-43)$$

where, as is usual,  $V$  is the interaction between the atoms.

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<sup>11</sup>See Anstee, S.D. and O'Mara, B.J. "An Investigation of Brueckner's Theory of Line Broadening with Application to the Sodium  $D$  Lines" *Monthly Notices of the Royal Astronomical Society* **253**, pg 549-560 (1991), or the earlier papers by O'Mara (see Bibliography).

<sup>12</sup>Unsöld, A. "Quantentheorie des Wasserstoffmoleküls und der Born-Landéschen Abstoßungskräfte" *Zeitschrift für Physik* **43**, pg 563-574 (1927).

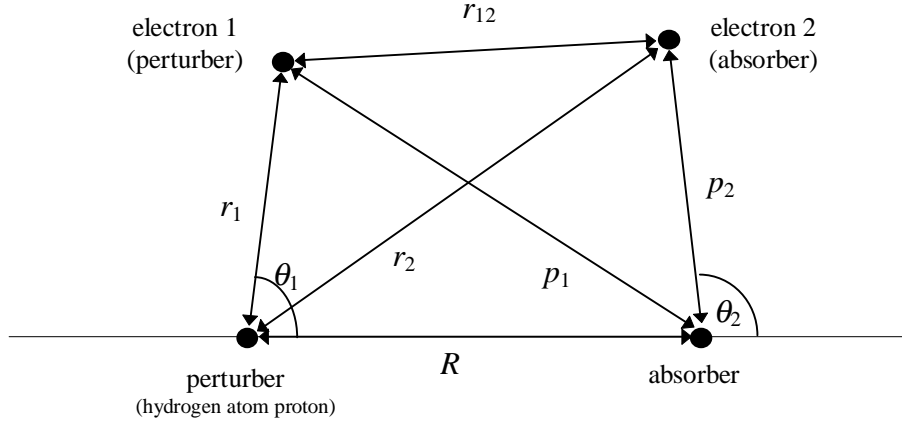


Figure 4-3: Geometry of Perturber-Absorber System

The interaction term  $V$  is given by

$$V = e \left( \frac{1}{R} + \frac{1}{r_{12}} + \frac{1}{r_2} + \bar{V}(p_1) \right) \quad (4-44)$$

where the absorbing atom is modelled as a single optically active electron outside a closed ionic core approximated by a Thomas-Fermi-Dirac ion (TFD ion). The potential of the TFD ionic core can be expressed in terms of a shielding function  $f(p_1)$  as

$$\bar{V}(p_1) = \frac{-1}{p_1} + f(p_1). \quad (4-45)$$

Far from the ionic core, the shielding function is zero, and approaches  $-(Z-1)/p_1$  as the nucleus of total charge  $Z$  is approached. For convenience, atomic units can be used wherein  $e = 1$ , distances are in Bohr radii and energies are in Hartree.

Rayleigh-Schrödinger perturbation theory will break down for small separations  $R$ , especially if  $R$  is close to or less than the distance to the potential minimum. At longer ranges, second-order perturbation theory will be sufficiently accurate.

The first-order interaction energy is

$$\Delta E_i^{(1)} = \langle \psi_i | V | \psi_i \rangle \quad (4-46)$$

and the second-order interaction energy is

$$\Delta E_i^{(2)} = \sum_{k \neq i} \frac{\langle \psi_i | V | \psi_k \rangle \langle \psi_k | V | \psi_i \rangle}{E_i^{(0)} - E_k^{(0)}}, \quad (4-47)$$

which, using Unsöld's approximation, reduces to

$$\Delta E_i^{(2)} = \frac{-1}{E_p} \left( \langle \psi_i | V^2 | \psi_i \rangle - (\Delta E_i^{(1)})^2 \right). \quad (4-48)$$

The atomic states here are for the combined system, but as virtually all hydrogen in the photosphere will be neutral and in the ground state, they will in practice only depend on the state  $i$  of the absorber. The first order energy is often assumed to be zero, as it is an exponentially decreasing function of  $R$ . At this point, given suitable wavefunctions for the atomic states, the potentials for the upper and lower states can be found.

To calculate the Lorentzian line profile, we can assume that the line width  $\Gamma$  and the shift  $\Delta\omega_0$  can be obtained from a complex cross-section  $\sigma(v)$ , where

$$\Gamma + i\Delta\omega_0 = N \int_0^{\infty} v W(v) \sigma(v) dv, \quad (4-49)$$

$v$  is the relative velocity, and  $W(v)$  is the probability distribution of relative velocities. In the impact approximation, the cross-section  $\sigma(v)$  is given in terms of the damping parameter  $\Pi(\rho, v)$  for a single collision as

$$\sigma(v) = 2\pi \int_0^{\infty} \Pi(\rho, v) \rho d\rho. \quad (4-50)$$

For perturbations by ground state hydrogen, the damping parameter becomes

$$\Pi(\rho, v) = \left( 1 - \frac{2l_{lower} + 1 \operatorname{tr}(S_{upper})}{2l_{upper} + 1 \operatorname{tr}(S_{lower})} \right). \quad (4-51)$$

The  $S$  matrices can then be calculated using the interaction energy of the system.

Once the damping parameter is known, the complex collision cross-section can be found using equation (4-50), and thus the line shift and width can be found. The line width can readily be found from equation (4-49) if the complex cross-section  $\sigma(v)$  is replaced by its real component. Then

$$\Gamma = N \int_0^{\infty} v W(v) \sigma(v) dv \quad (4-52)$$

where  $\sigma(v)$  is the real collision cross-section. As  $\sigma(v)$  is a slowly varying function of the relative velocity  $v$ , this expression for the line width  $\Gamma$  can be well approximated by

$$\Gamma = N \bar{v} \sigma(\bar{v}) \quad (4-53)$$

in terms of the average relative velocity (as given by equation (4-26) ). For convenience, the cross-sections calculated using this theory can be expressed in terms of the cross-section at a reference velocity and an interpolation constant  $\alpha$  such that

$$\sigma(v) = \sigma(v = 10 \text{ kms}^{-1}) \left( \frac{v}{10 \text{ kms}^{-1}} \right)^{-\alpha} . \quad (4-54)$$

Given these  $\sigma_{10}$  and  $\alpha$  values, calculation of the line width is straightforward.<sup>13</sup>

The relationship between this cross-section and the usual van der Waals constant  $C_6$  can be found from equation (4-37), giving

$$C_6 = \frac{\bar{v}}{186} \sigma(\bar{v})^{5/2} \quad (4-55)$$

where the mean velocity is in c.g.s. units. This can be used to compare results.

Damping constants given by the Brueckner-O'Mara theory appear to be of reasonable accuracy. The lack of accurate experimental results hinders comparison, but, particularly in view of the inaccuracy of other common methods of calculating damping constants, the theoretical damping constants obtained in this way may well be the best available.<sup>14</sup>

#### 4.6: Statistical Broadening Theory

Impact broadening theory does not adequately account for perturbations caused by multiple particles simultaneously. Statistical broadening theory (the quasi-static approximation) is an attempt to deal with this shortcoming. The usual starting point is equation (4-21), in wavelength units giving

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<sup>13</sup>Software for the calculation of these constants was supplied by J.E. Ross (Physics Department, The University of Queensland).

<sup>14</sup>Anstee and O'Mara compare various theoretical and experimental results for the widths of the sodium *D* lines in Anstee, S.D. and O'Mara, B.J. "An Investigation of Brueckner's Theory of Line Broadening with Application to the Sodium *D* Lines" *Monthly Notices of the Royal Astronomical Society* **253**, pg 549-560 (1991). See also Milford, P.N. "Line Intensity Ratios and the Solar Abundance of Iron" PhD Thesis, The University of Queensland (1987). Recent work by O'Mara confirms the general accuracy of the theory.

$$\Delta\lambda = \frac{C_p}{r^p}. \quad (4-56)$$

Each perturber is then assumed to produce a  $1/r^p$  field, and the probability distribution of the combined field is then found.<sup>15</sup> This standard treatment is incorrect; while it will produce a reasonable result, as it will correctly give the effect of the nearest perturber, it will not correctly predict combined effects. (Consider the difference between combining two inverse square fields and finding the square of the magnitude and combining two  $1/r^4$  fields. Note that  $|\mathbf{a} + \mathbf{b}|^2 \neq |\mathbf{a}|^2 \hat{\mathbf{a}} + |\mathbf{b}|^2 \hat{\mathbf{b}}$  in most cases.) There is no reason not to attempt a correct calculation of the distribution of the wavelength shift.

#### 4.6.1: The Stark Effect and Statistical Broadening Theory

Whether or not the perturbing particle is an ion, electron or a neutral atom, unless it is very close to the absorber, the interaction is caused by the effect of the electric field of the perturber at the absorber. Thus, the interactions are all due to the Stark effect, even those normally labelled as van der Waals broadening ( $p = 6$ ). If we know the probability distribution for the magnitude of the electric field,  $W(E)dE$ , the probability distribution for the wavelength shift  $W(\Delta\lambda)d\Delta\lambda$  can be found. For a quadratic Stark effect<sup>16</sup>, the wavelength shift is related to the field by

$$\Delta\lambda = C_s E^2 \quad (4-57)$$

where  $C_s$  is the constant of proportionality for the quadratic Stark effect for the transition in question. Then, since

$$d\Delta\lambda = 2C_s E dE, \quad (4-58)$$

the probability distribution is given by

$$W(\Delta\lambda)d\Delta\lambda = \frac{W(E)}{2C_s E} d\Delta\lambda. \quad (4-59)$$

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<sup>15</sup>See, for example, pg 265 in Mihalas, D. "Stellar Atmospheres" Freeman (1970). Here, the procedure is adapted from a calculation by Chandrasekhar of the probability distribution of the gravitational field caused by an infinite number of identical randomly distributed stars, in Chandrasekhar, S. "Stochastic Methods in Astrophysics", *Reviews of Modern Physics* **15**, pg 1 (1943). The calculation of an electric field distribution is similar.

<sup>16</sup>Here we will ignore the fact that the external electric field at our absorber is not spatially uniform, and thus assume that the shift can be given simply in terms of the electric field magnitude.

The difficulty arises when we try to calculate  $W(E)dE$ . If we assume that we have an infinite number of perturbers uniformly distributed, we obtain a Holtsmark distribution<sup>17</sup>. Distributions of this form have been calculated, but generally only for either charged particles (ions and electrons) or dipoles (neutral atoms) individually, rather than both simultaneously. Also, neutral atoms are usually treated as producing a pure  $1/r^3$  dipole field, rather than considering the effects of variation in the dipole moment and direction of the dipole, resulting in an incorrect distribution (but correct mean behaviour).

The approximation of a perturbing neutral atom as a dipole will only be adequate at large separations, but at closer separations, the quasi-static approximation will fail anyway, so it should prove adequate for the purpose.

#### 4.6.2: Holtsmark Theory

The distribution of the particles will not be random (for example, the position of charged particles will be affected by the electric field), so we must also determine how good an approximation the Holtsmark distribution will actually be. Field distributions taking the non-random distribution of particles have been calculated<sup>18</sup>, but, as under the conditions in the sun, the Holtsmark distribution, which will be easier to calculate, may be a quite good approximation.

A plasma can be characterised by its Debye length which is the effective distance over which the electric field of a particle interacts with other particles before being cancelled out by the fields of shielding particles. Since ions can be shielded by both electrons and other ions, the Debye length can be found by considering a total charged particle density of  $2N_e$  (assuming that all of the ions have a single positive charge) which gives a Debye length of

$$\left(\lambda_{\text{Debye}}\right)_{\text{ions}} = \sqrt{\frac{kT}{8\pi N_e e^2}} \quad (4-60)$$

where  $e$  is the charge of an electron. This will also be the Debye length for dipoles. Only electrons will act to shield the field due to other electrons (due to their higher speeds), so the Debye length for electrons is given by

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<sup>17</sup>Holtsmark, J. "Über die Verbreiterung von Spektrallinien" *Annalen der Physik* **58**, pg 577 (1919)

<sup>18</sup>See Mozer, B. and Baranger, M. "Electric Field Distributions in an Ionized Gas. II", in *Physical Review* **118**, pg 626 (1960) where they calculate the field distribution for a completely ionised gas under varying conditions.

$$\left(\lambda_{\text{Debye}}\right)_{\text{electrons}} = \sqrt{\frac{kT}{2\pi N_e e^2}} \quad (4-61)$$

The total number of ions or dipoles of number density  $N_i$  within a Debye sphere (a sphere of radius equal to the Debye length), being the number of particles which contribute to the field at a point, is given by

$$\left(n_{\text{Debye}}\right)_i = \frac{N_i}{12\sqrt{2\pi}} \left(\frac{kT}{N_e e^2}\right)^{\frac{3}{2}} \quad (4-62)$$

and the number of electrons within a Debye sphere is

$$\left(n_{\text{Debye}}\right)_{\text{electrons}} = \frac{1}{6\sqrt{\pi N_e}} \left(\frac{kT}{e^2}\right)^{\frac{3}{2}} \quad (4-63)$$

The number of particles in the Debye sphere at various depths in the photosphere can then be found. (See table 4-2.)

Table 4-2: The number of particles within a Debye sphere for various species of particles (using Holweger-Müller atmosphere).

Optical Depth $\tau_o$	Hydrogen Atoms	Charged Particles	Electrons	Ions
0.0001	$3.4 \times 10^6$	1400	1040	370
0.01	$1.2 \times 10^6$	470	350	120
0.10	$6.0 \times 10^5$	280	200	72
0.32	$3.2 \times 10^5$	200	150	52
0.63	$1.2 \times 10^5$	150	110	38
1.0	$4.8 \times 10^4$	110	82	29
1.6	$2.2 \times 10^4$	87	64	23
2.0	$1.2 \times 10^4$	75	56	20
3.2	$4.1 \times 10^3$	56	41	15
4.0	$2.5 \times 10^3$	49	36	13
10.0	$9.5 \times 10^2$	37	28	10

Since there are a large number of hydrogen atoms within a Debye sphere, the Holtsmark distribution will be a very good approximation for the dipole field distribution, and the number of charged particles is also high enough so that the Holtsmark distribution will be a reasonable approximation.

### 4.6.3: Holtmark Theory Revisited

If we consider a region of the photosphere to be uniform and effectively infinite, with a charged particle number density of  $N_c$  and a (hydrogen atom) dipole density of  $N_d$ , we can determine the probability distribution of the field due to both charged particles and dipoles.<sup>19</sup> See table 4-3 for a summary of notation used in this calculation.

Table 4-3: Summary of notation used in calculation of  
Holtmark distribution.

Symbol	Meaning of Symbol
$W(\eta)d\eta$	Probability distribution function for $\eta$
$\mathbf{E}_i = \mathbf{E}_i(\mathbf{r}_i)$	Electric field due to $i$ th charged particle
$\mathbf{r}_i$	Position vector for $i$ th charged particle
$W(\mathbf{E}_i)d\mathbf{E}_i$	Field distribution for a charged particle
$\mathbf{E}_j = \mathbf{E}_j(\mathbf{r}_j, \mathbf{l}_j)$	Electric field due to $j$ th dipole
$\mathbf{r}_j$	Position vector for $j$ th dipole
$\mathbf{l}_j$	Dipole separation vector for $j$ th dipole
$\mathbf{E}$	Total electric field
$W(\mathbf{E})d\mathbf{E}$	Distribution for total electric field
$E$	Magnitude of total electric field
$W(E)dE$	Distribution for magnitude of total field

First we will consider the field due to  $n_c$  identical charged particles and  $n_d$  dipoles, each with an identical distribution for their dipole separation vectors. (The dipole separation vector gives the direction of the dipole moment and the distance between the two charges comprising the dipole.) If we know the field distributions for the charged particles and the dipoles, we can then write the total field distribution as the convolution of all of the individual particle distributions:

$$W(\mathbf{E}) = W(\mathbf{E}_{i=1}) \otimes \dots \otimes W(\mathbf{E}_{i=n_c}) \otimes W(\mathbf{E}_{j=1}) \otimes \dots \otimes W(\mathbf{E}_{j=n_d}). \quad (4-64)$$

As there are a great many particles, it is not possible to calculate this directly, but we can rewrite this convolution as a product in a Fourier domain as

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<sup>19</sup>This calculation follows Chandrasekhar (in Chandrasekhar, S. "Stochastic Methods in Physics and Astronomy" *Reviews of Modern Physics* **15**, pg 1 (1943) ), but deals simultaneously with both kinds of particles (charged particles and dipoles).

$$W(\mathbf{E}) = \frac{1}{8\pi^3} \iiint_{\text{all space}} [A_i(\boldsymbol{\rho})]^{n_c} [A_j(\boldsymbol{\rho})]^{n_d} e^{-i\mathbf{r}\cdot\mathbf{E}} d\boldsymbol{\rho} \quad (4-65)$$

where  $A_i(\boldsymbol{\rho})$  and  $A_j(\boldsymbol{\rho})$  are the Fourier transforms of  $W(\mathbf{E}_i)$  and  $W(\mathbf{E}_j)$ .<sup>20</sup> The Fourier transform for charged particles is given by

$$A_i(\boldsymbol{\rho}) = \iiint_{\text{all fields}} W(\mathbf{E}_i) e^{i\boldsymbol{\rho}\cdot\mathbf{E}_i} d\mathbf{E}_i, \quad (4-66)$$

or, in terms of  $W(\mathbf{r}_i)$  and  $\mathbf{r}_i$  instead of  $W(\mathbf{E}_i)$  and  $\mathbf{E}_i$ :

$$A_i(\boldsymbol{\rho}) = \iiint_{\text{all space}} W(\mathbf{r}_i) e^{i\boldsymbol{\rho}\cdot\mathbf{E}_i(\mathbf{r}_i)} d\mathbf{r}_i. \quad (4-67)$$

If the particles are uniformly distributed over a volume  $V$ , then  $W(\mathbf{r}_i) = \frac{1}{V}$  and

$$A_i(\boldsymbol{\rho}) = \frac{1}{V} \iiint_{\text{all space}} e^{i\boldsymbol{\rho}\cdot\mathbf{E}_i(\mathbf{r}_i)} d\mathbf{r}_i. \quad (4-68)$$

Similarly, for dipoles, the Fourier transform is

$$A_j(\boldsymbol{\rho}) = \iiint_{\text{all fields}} W(\mathbf{E}_j) e^{i\boldsymbol{\rho}\cdot\mathbf{E}_j(\mathbf{r}_j, \mathbf{l}_j)} d\mathbf{E}_j \quad (4-69)$$

and we can write this in terms of  $W(\mathbf{r}_j)$ ,  $\mathbf{r}_j$ ,  $W(\mathbf{l}_j)$  and  $\mathbf{l}_j$ :

$$A_j(\boldsymbol{\rho}) = \iiint_{\text{all space}} \iiint_{\text{all dipole vectors}} W(\mathbf{r}_j) W(\mathbf{l}_j) e^{i\boldsymbol{\rho}\cdot\mathbf{E}_j(\mathbf{r}_j, \mathbf{l}_j)} d\mathbf{l}_j d\mathbf{r}_j. \quad (4-70)$$

Again,  $W(\mathbf{r}_j) = \frac{1}{V}$ , and  $W(\mathbf{l}_j) = \frac{1}{\pi a_o^3} e^{-\frac{2l}{a_o}}$  (where  $a_o$  is the Bohr radius),<sup>21</sup> so

$$A_j(\boldsymbol{\rho}) = \frac{1}{V} \iiint_{\text{all space}} \iiint_{\text{all dipole vectors}} \frac{1}{\pi a_o^3} e^{-\frac{2l}{a_o}} e^{i\boldsymbol{\rho}\cdot\mathbf{E}_j(\mathbf{r}_j, \mathbf{l}_j)} d\mathbf{l}_j d\mathbf{r}_j. \quad (4-71)$$

Using these expressions for the Fourier transforms,  $W(\mathbf{E})$  is given by

<sup>20</sup>Chandrasekhar (in Chandrasekhar, S. "Stochastic Methods in Physics and Astronomy" *Reviews of Modern Physics* **15**, pg 1 (1943)) arrives at this point via a different route, by starting with the sum of the probabilities of all configurations giving a particular value for the total field, and then transforming this sum into an integral over all space by using the discontinuous integral of Dirichlet, and thus obtaining equation (5-25) for the species of perturber in question.

<sup>21</sup>This result, the probability distribution for the location of an electron in the ground state of a

hydrogen atom may be more familiar in the form  $W(l)dl = \frac{4}{a_o^3} l^2 e^{-\frac{2l}{a_o}} dl$  where  $d\mathbf{l}$  is converted to

polar coordinates and, as there is no angle dependence, integrated over all angles. This is used for  $W(\mathbf{l})$  as the only atom we are considering as a strong dipole is hydrogen, and almost all hydrogen atoms will be in the ground state due to the large separation between the ground state energy and the energy of the first excited state.

$$W(\mathbf{E}) = \frac{1}{8\pi^3} \iiint A(\boldsymbol{\rho}) e^{-i\boldsymbol{\rho}\cdot\mathbf{E}} d\boldsymbol{\rho} \quad (4-72)$$

where

$$A(\boldsymbol{\rho}) = \left[ \frac{1}{V} \iiint e^{i\boldsymbol{\rho}\cdot\mathbf{E}_i(\mathbf{r}_i)} d\mathbf{r}_i \right]^{N_c V} \left[ \frac{1}{V} \iiint \iiint \frac{e^{-\frac{2l}{a_o}}}{\pi a_o^3} e^{i\boldsymbol{\rho}\cdot\mathbf{E}_j(\mathbf{r}_j, \mathbf{l}_j)} d\mathbf{l}_j d\mathbf{r}_j \right]^{N_d V} \quad (4-73)$$

where  $N_c$  and  $N_d$  are the number densities of charged particles and dipoles, so that the total number of charged particles is given by  $n_c = N_c V$ , and the total number of dipoles is given by  $n_d = N_d V$ .

Since we are considering a very large volume with an effectively infinite number of particles, we can consider the limit when  $V$  approaches infinity. Keeping in mind that

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{x}{n} \right)^n = e^x \quad (4-74)$$

we can rewrite equation (4-73) as

$$A(\boldsymbol{\rho}) = \left[ 1 + \frac{-N_c \iiint 1 - e^{i\boldsymbol{\rho}\cdot\mathbf{E}_i(\mathbf{r}_i)} d\mathbf{r}_i}{N_c V} \right]^{N_c V} \times \left[ 1 + \frac{-N_d \frac{1}{\pi a_o^3} \iiint \iiint e^{-\frac{2l}{a_o}} \left( 1 - e^{i\boldsymbol{\rho}\cdot\mathbf{E}_j(\mathbf{r}_j, \mathbf{l}_j)} \right) d\mathbf{l}_j d\mathbf{r}_j}{N_d V} \right]^{N_d V} \quad (4-75)$$

which, using equation (4-74), gives

$$A(\boldsymbol{\rho}) = e^{-N_c D_c(\boldsymbol{\rho}) - N_d D_d(\boldsymbol{\rho})} \quad (4-76)$$

where  $D_c$  and  $D_d$  are given by

$$D_c(\boldsymbol{\rho}) = \iiint_{\text{all space}} 1 - e^{i\boldsymbol{\rho}\cdot\mathbf{E}_i(\mathbf{r}_i)} d\mathbf{r}_i \quad (4-77)$$

and

$$D_d(\boldsymbol{\rho}) = \frac{1}{\pi a_o^3} \iiint_{\text{all space}} \iiint_{\text{all dipole vectors}} e^{-\frac{2l}{a_o}} \left( 1 - e^{i\boldsymbol{\rho}\cdot\mathbf{E}_j(\mathbf{r}_j, \mathbf{l}_j)} \right) d\mathbf{l}_j d\mathbf{r}_j. \quad (4-78)$$

If these expressions can be calculated, then  $W(\mathbf{E})$  can be found.

If equation (4-77) is rewritten in polar coordinates, with  $\boldsymbol{\rho}$  directed along the z-axis, we then have

$$D_c(\boldsymbol{\rho}) = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \int_{r=0}^{\infty} (1 - e^{i\rho E_i \cos\phi}) r^2 \sin\phi dr d\phi d\theta. \quad (4-79)$$

The magnitude of the electric field is given by

$$E_i = \frac{C_c}{r^2} \quad (4-80)$$

where  $C_c$  is the field strength constant. (Equal to the electronic charge  $e$  in this case.)

Then

$$D_c(\boldsymbol{\rho}) = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \int_{r=0}^{\infty} (1 - e^{-\frac{i\rho C_c \cos\phi}{r^2}}) r^2 \sin\phi \, dr \, d\phi \, d\theta. \quad (4-81)$$

Integrating with respect to  $\theta$  gives

$$D_c(\boldsymbol{\rho}) = 2\pi \int_{\phi=0}^{\pi} \int_{r=0}^{\infty} (1 - e^{-\frac{i\rho C_c \cos\phi}{r^2}}) r^2 \sin\phi \, dr \, d\phi. \quad (4-82)$$

If we substitute  $y = \cos\phi$ ,  $dy = -\sin\phi \, d\phi$ , we obtain

$$D_c(\boldsymbol{\rho}) = 2\pi \int_{y=-1}^1 \int_{r=0}^{\infty} (1 - e^{-\frac{i\rho C_c y}{r^2}}) r^2 \, dr \, dy \quad (4-83)$$

which can be (since  $\sin \frac{\rho C_c y}{r^2}$  is odd w.r.t.  $y$ ) written as

$$D_c(\boldsymbol{\rho}) = 2\pi \int_{y=-1}^1 \int_{r=0}^{\infty} (1 - \cos \frac{\rho C_c y}{r^2}) r^2 \, dr \, dy \quad (4-84)$$

which can then be integrated to give

$$D_c(\boldsymbol{\rho}) = 4\pi \int_0^{\infty} \left( 1 - \frac{\sin(\rho C_c / r^2)}{\rho C_c / r^2} \right) r^2 \, dr. \quad (4-85)$$

Substituting  $z = \frac{\rho C_c}{r^2}$  and  $r^2 \, dr = -\frac{(\rho C_c)^{\frac{3}{2}}}{2z^{\frac{5}{2}}} \, dz$ , we obtain

$$\begin{aligned} D_c(\boldsymbol{\rho}) &= 4\pi \int_0^{\infty} \left( 1 - \frac{\sin z}{z} \right) \frac{(\rho C_c)^{\frac{3}{2}}}{2z^{\frac{5}{2}}} \, dz \\ &= 2\pi (\rho C_c)^{\frac{3}{2}} \int_0^{\infty} \frac{z - \sin z}{z^{\frac{7}{2}}} \, dz \end{aligned} \quad (4-86)$$

which can be evaluated to give

$$\begin{aligned} D_c(\boldsymbol{\rho}) &= 2\pi (\rho C_c)^{\frac{3}{2}} \frac{4\sqrt{2}\pi}{15} \\ &= (\gamma_c \rho C_c)^{\frac{3}{2}} \end{aligned} \quad (4-87)$$

where

$$\gamma_c = \left( \frac{8\sqrt{2}\pi^{\frac{3}{2}}}{15} \right)^{\frac{2}{3}} \cong 2.6031. \quad (4-88)$$

Similarly, we can find a simple expression for  $D_a$ . If we rewrite equation (4-78) in polar coordinates  $\theta$ ,  $\phi$ , and  $r$  for the position vector  $\mathbf{r}_j$ , and  $\xi$ ,  $\psi$ , and  $l$  for the dipole vector  $\mathbf{l}_j$ , we have

$$D_d(\rho) = \frac{1}{\pi a_o^3} \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \int_{r=0}^{\infty} \int_{\xi=0}^{2\pi} \int_{\psi=0}^{\pi} \int_{l=0}^{\infty} e^{-\frac{2l}{a_o}} \left(1 - e^{i\rho E_j \cos \phi}\right) \times l^2 \sin \psi dl d\psi d\xi r^2 \sin \phi dr d\phi d\theta. \quad (4-89)$$

The magnitude of the dipole field is given by

$$E_j = \frac{C_d l}{r^3} \sqrt{1 + 3 \cos^2 \psi} \quad (4-90)$$

where  $C_d l$  is the dipole moment. (Here,  $C_d$  is the electronic charge  $e$  and  $l$  is separation between the proton and the electron.) If we integrate with respect to  $\theta$  and  $\xi$  and use equation (4-90) for  $E_j$  we obtain

$$D_d(\rho) = \frac{4\pi}{a_o^3} \int_{\phi=0}^{\pi} \int_{r=0}^{\infty} \int_{\psi=0}^{\pi} \int_{l=0}^{\infty} e^{-\frac{2l}{a_o}} \left(1 - e^{i \frac{\rho C_d}{r^3} \sqrt{1+3\cos^2 \psi} \cos \phi}\right) \times l^2 \sin \psi dl d\psi r^2 \sin \phi dr d\phi. \quad (4-91)$$

Substituting  $x = \cos \psi$ ,  $dx = \sin \psi d\psi$ ,  $y = \cos \phi$  and  $dy = \sin \phi d\phi$ , this gives

$$D_d(\rho) = \frac{4\pi}{a_o^3} \int_{y=-1}^1 \int_{r=0}^{\infty} \int_{x=-1}^1 \int_{l=0}^{\infty} e^{-\frac{2l}{a_o}} \left(1 - e^{i \frac{\rho C_d l y}{r^3} \sqrt{1+3x^2}}\right) l^2 r^2 dl dx dr dy. \quad (4-92)$$

As before, the sine term in the  $e^{i\dots}$  can be dropped, and the expression integrated in  $y$ :

$$D_d(\rho) = \frac{8\pi}{a_o^3} \int_{r=0}^{\infty} \int_{x=-1}^1 \int_{l=0}^{\infty} e^{-\frac{2l}{a_o}} \left(1 - \frac{\sin\left(\frac{\rho C_d l \sqrt{1+3x^2}}{r^3}\right)}{\frac{\rho C_d l \sqrt{1+3x^2}}{r^3}}\right) l^2 r^2 dl dx dr. \quad (4-93)$$

If we now substitute  $z = \frac{\rho C_d l \sqrt{1+3x^2}}{r^3}$  and  $r^2 dr = \frac{-\rho C_d l \sqrt{1+3x^2}}{3z^2}$  this becomes

$$\begin{aligned} D_d(\rho) &= \frac{8\pi \rho C_d}{3a_o^3} \int_{z=0}^{\infty} \int_{x=-1}^1 \int_{l=0}^{\infty} l^3 e^{-\frac{2l}{a_o}} \sqrt{1+3x^2} \left(\frac{z - \sin z}{z^3}\right) dl dx dz \\ &= \frac{2\pi^2 \rho C_d}{3a_o^3} \int_{x=-1}^1 \int_{l=0}^{\infty} l^3 e^{-\frac{2l}{a_o}} \sqrt{1+3x^2} dl dx \\ &= \frac{\pi^2 a_o \rho C_d}{4} \int_{-1}^1 \sqrt{1+3x^2} dx \\ &= \frac{\pi^2 a_o \rho C_d}{4} \left(2 + \frac{\ln(2+\sqrt{3})}{\sqrt{3}}\right) \\ &= \gamma_d a_o \rho C_d \end{aligned} \quad (4-94)$$

where

$$\gamma_d = \frac{\pi^2}{4} \left(2 + \frac{\ln(2+\sqrt{3})}{\sqrt{3}}\right) \cong 6.8109. \quad (4-95)$$

The values that have been obtained for  $D_c$  and  $D_d$  can be substituted into equation (4-76) to give

$$A(\rho) = e^{-N_c (\gamma_c C_c \rho)^{\frac{3}{2}} - N_d (\gamma_d a_o C_d \rho)} \quad (4-96)$$

We can now find  $W(\mathbf{E})$ :

$$W(\mathbf{E}) = \frac{1}{8\pi^3} \iiint e^{-N_c(\gamma_c C_c \rho)^{\frac{3}{2}} - N_d(\gamma_d a_o C_d \rho)} e^{-i\mathbf{p}\cdot\mathbf{E}} d\mathbf{p}, \quad (4-97)$$

or, in polar coordinates,

$$W(\mathbf{E}) = \frac{1}{8\pi^3} \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \int_{\rho=0}^{\infty} e^{-N_c(\gamma_c C_c \rho)^{\frac{3}{2}} - N_d(\gamma_d a_o C_d \rho)} e^{-i\rho E \cos\phi} \rho^2 \sin\phi d\rho d\phi d\theta. \quad (4-98)$$

As before, we can now integrate over  $\theta$ , giving

$$W(\mathbf{E}) = \frac{1}{4\pi^2} \int_{\phi=0}^{\pi} \int_{\rho=0}^{\infty} e^{-N_c(\gamma_c C_c \rho)^{\frac{3}{2}} - N_d(\gamma_d a_o C_d \rho)} e^{-i\rho E \cos\phi} \rho^2 \sin\phi d\rho d\phi. \quad (4-99)$$

Substituting  $z = \cos\phi$  and  $dz = -\sin\phi d\phi$ , and dropping the sine term from the complex exponential,

$$W(\mathbf{E}) = \frac{1}{4\pi^2} \int_{z=-1}^1 \int_{\rho=0}^{\infty} e^{-N_c(\gamma_c C_c \rho)^{\frac{3}{2}} - N_d(\gamma_d a_o C_d \rho)} \cos(-\rho E z) \rho^2 d\rho dz. \quad (4-100)$$

This can now be integrated to give

$$W(\mathbf{E}) = \frac{1}{2\pi^2 E} \int_0^{\infty} e^{-N_c(\gamma_c C_c \rho)^{\frac{3}{2}} - N_d(\gamma_d a_o C_d \rho)} \rho \sin(\rho E) d\rho, \quad (4-101)$$

into which we can now substitute  $x = \rho E$ , giving

$$W(\mathbf{E}) = \frac{1}{2\pi^2 E^3} \int_0^{\infty} e^{-N_c \left(\frac{\gamma_c C_c x}{E}\right)^{\frac{3}{2}} - N_d \left(\frac{\gamma_d a_o C_d x}{E}\right)} x \sin x dx. \quad (4-102)$$

We now define a normal charged particle field as

$$F_c = \gamma_c C_c N_c^{\frac{2}{3}} \cong 1.2502 \times 10^{-9} N_c^{\frac{2}{3}} \quad (4-103)$$

and a normal dipole field as

$$F_d = \gamma_d a_o C_d N_d \cong 1.7304 \times 10^{-17} N_d, \quad (4-104)$$

and  $W(\mathbf{E})$  in terms of these normal fields is then

$$W(\mathbf{E}) = \frac{1}{2\pi^2 E^3} \int_0^{\infty} e^{-\left[\left(\frac{F_c x}{E}\right)^{\frac{3}{2}} + \left(\frac{F_d x}{E}\right)\right]} x \sin x dx. \quad (4-105)$$

We do not actually want  $W(\mathbf{E})$ , the electric field distribution function, but rather  $W(E)$ , the distribution function for the magnitude of the electric field. So, as  $W(E) = 4\pi^2 E^2 W(\mathbf{E})$ ,

$$W(E) = \frac{2}{\pi E} \int_0^{\infty} e^{-\left[\left(\frac{F_c x}{E}\right)^{\frac{3}{2}} + \left(\frac{F_d x}{E}\right)\right]} x \sin x dx \quad (4-106)$$

or, if preferred, with  $y = x/E$ ,

$$W(E) = \frac{2E}{\pi} \int_0^{\infty} e^{-\left[(F_c y)^{\frac{3}{2}} + F_d y\right]} y \sin(Ey) dy. \quad (4-107)$$

If we define a normal field ratio  $\alpha$  as

$$\alpha = \frac{F_d}{F_c} \quad (4-108)$$

and measure the field in units of the normal charged particle field strength (giving the distribution in as close as possible to the standard notation<sup>22</sup>),

$$\beta = \frac{E}{F_c}. \quad (4-109)$$

Then  $W(\beta) = F_c W(E)$ , giving us

$$W(\beta) = \frac{2}{\pi\beta} \int_0^\infty e^{-\left[\left(\frac{x}{\beta}\right)^{\frac{3}{2}} + \frac{\alpha x}{\beta}\right]} x \sin x \, dx \quad (4-110)$$

or

$$W(\beta) = \frac{2\beta}{\pi} \int_0^\infty e^{-\left[y^{\frac{3}{2}} + \alpha y\right]} y \sin(\beta y) \, dy. \quad (4-111)$$

These integrals (equations (4-108), (4-109), (4-110) and (4-111)) can then be numerically integrated to determine the field distribution for a set of given conditions from which the normal field strengths are determined. (See table 4-4 for normal field strengths in the photosphere.)

Table 4-4: Normal Fields in the Photosphere

Optical Depth $\tau_0$	$F_c$ (esu)	$F_d$ (esu)	$\alpha$
0.0001	0.0554	0.0212	0.382
0.001	0.128	0.0764	0.597
0.01	0.287	0.264	0.921
0.10	0.697	0.859	1.23
0.32	1.24	1.46	1.18
0.63	2.23	1.82	0.819
1.0	3.62	2.00	0.553
1.6	5.61	2.27	0.407
2.0	7.28	2.18	0.300
3.2	12.5	2.16	0.174
4.0	15.9	2.20	0.139
5.0	19.1	2.21	0.115
10.0	26.3	2.31	0.0879

<sup>22</sup>If we only considered charged particles or dipoles individually, this would actually give the standard form, but here we have two separate normal fields and no simple method to measure  $E$  in terms of both of them.

From these results, we can see that the fields caused by charged particles and dipoles are both important; both must be taken into account for an accurate treatment of broadening.

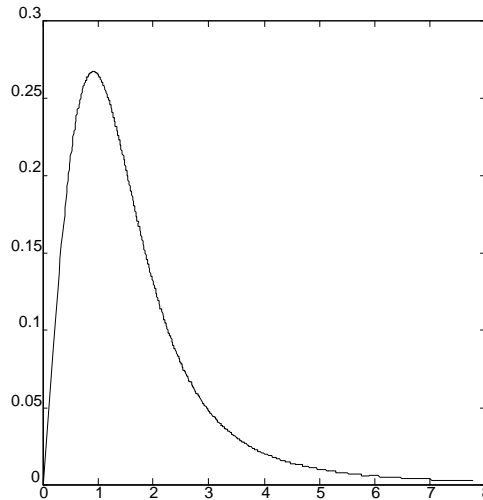
The simple classical treatment of the hydrogen atom dipole used here will be an adequate approximation when estimating the importance of quasi-static damping. If accurate numerical results are needed, the appropriate integrals can be replaced by sums over discrete states.

#### **4.7: Spectral Lines and the Holtmark Distribution**

The distribution for the line shift will be given by equation (4-59), giving

$$W(\Delta\lambda)d\Delta\lambda = \left\{ \frac{1}{\pi C_s} \int_0^{\infty} e^{-\left[(F_c y)^2 + F_d y\right]} y \sin\left(y \sqrt{\frac{\Delta\lambda}{C_s}}\right) dy \right\} d\Delta\lambda \quad (4-112)$$

using equation (4-107), or a similar expression using any of the other three forms of the field distribution. A sample Holtmark distribution is shown in figure 4-4.



**Figure 4-4: The Holtmark Distribution**

##### **4.7.1: Combining the Impact and Quasi-static Approximations**

For photospheric damping, quasi-static damping is usually ignored, as the contribution to the width will be negligible. The impact approximation alone is used for the damping, and given suitable interaction constants, gives an accurate result for

the width of the Lorentzian profile. The quasi-static damping, however, is strongly asymmetric even if it contributes negligibly to the overall line width.

A suitable method to find the combined effect of impact and quasi-static broadening is to use the impact approximation to determine the Lorentzian width  $\Gamma$  and the quasi-static theory to calculate the line shift distribution, as given by equation (4-112). The result will then be a convolution between the two profiles.

A quick examination of such combinations (see figure 4-5) shows us that the quasi-static damping can contribute significantly to the asymmetry of the line while effecting only minor changes in the line width.

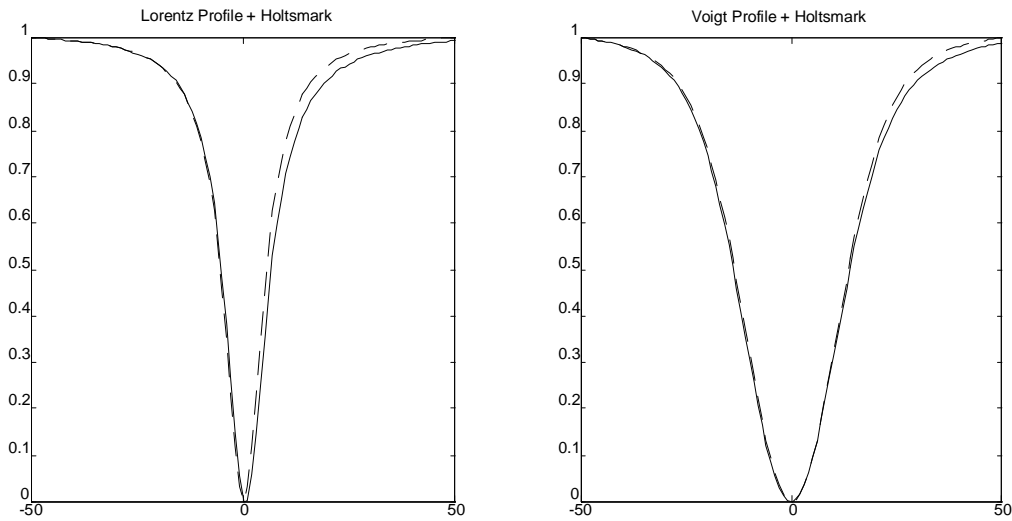


Figure 4-5: Combination of Holtzmark profile and Lorentz and Voigt Profiles

The asymmetry caused by the damping is also the same general type of asymmetry that is seen in photospheric absorption lines. Thus, it is important to be able to accurately estimate the contribution to the line profile due to quasi-static damping.

#### 4.7.2: The Quasi-Static Contribution

The quasi-static and impact broadening are related by the same interactions being responsible for both. Thus, in the case of the photosphere, we can determine the relative importance of the quasi-static contribution.

In order to investigate the importance of quasi-static damping, we must be able to compare the Stark coefficient  $C_s$  used in the quasi-static calculations with the

damping interaction constants  $C_4$  and  $C_6$  used in the impact approximation calculations. The mean value of the magnitude of the dipole field given by equation (4-90) when averaged over all orientations and all dipole separations will be approximately

$$\bar{E} = \frac{ea_0\sqrt{2}}{r^3}. \quad (4-113)$$

This gives a wavelength shift of

$$\Delta\lambda = \frac{2C_s e^2 a_0^2}{r^6}. \quad (4-114)$$

The corresponding shift in angular frequency is

$$\Delta\omega = \frac{4\pi C_s e^2 a_0^2 c}{\lambda^2 r^6} \quad (4-115)$$

and the damping constant  $C_6$  is thus

$$C_6 = \frac{4\pi e^2 a_0^2 c}{\lambda^2} C_s. \quad (4-116)$$

Similarly,

$$C_4 = \frac{2\pi e^2 c}{\lambda^2} C_s \quad (4-117)$$

and

$$C_6 = 2a_0^2 C_4. \quad (4-118)$$

In order to compare the effects of the Lorentzian impact broadening and the Holtsmarkian quasi-static broadening, we can assume a (fictitious) Fe I spectral line at  $5000\text{\AA}$  with  $C_6 = 5.0 \times 10^{-32}$  (of the same order of magnitude as  $C_6$  for the sodium D lines). Then,  $C_4 = 8.9 \times 10^{-16}$  and  $C_s = 5.2 \times 10^{-17}$  (in standard c.g.s. units).

The impact damping can then be calculated for various heights within the photosphere (see table 4-5).

Table 4-5: Impact Damping Constants

Optical Depth $\tau_0$	Hydrogen $\Gamma_6$	Ions and Electrons $\Gamma_4$	Total Damping $\Gamma$	Minimum Significant Wavelength Shift $\Delta\lambda$ ( $\text{\AA}$ )
0.0001	$1.2 \times 10^7$	$5.0 \times 10^4$	$1.2 \times 10^7$	$1.6 \times 10^{-5}$
1.0	$1.4 \times 10^9$	$3.2 \times 10^7$	$1.4 \times 10^9$	$1.9 \times 10^{-3}$
10	$1.6 \times 10^9$	$6.4 \times 10^8$	$2.3 \times 10^9$	$3.1 \times 10^{-3}$

The importance of collisions with neutral hydrogen can be seen in table 4-5, as they clearly provide the greatest contribution to the Lorentzian line width.

The Holtzmark distributions at these heights can be calculated numerically. The electric fields needed to give wavelength shifts equal to these Lorentzian line widths are shown in table 4-6.

Table 4-6: Electric Fields Required for Shifts

Line width (Å)	Field for shift (e.s.u.)
$1.6 \times 10^{-5}$	$5.5 \times 10^5$
$1.9 \times 10^{-3}$	$6.0 \times 10^6$
$3.1 \times 10^{-3}$	$7.7 \times 10^6$

The Holtzmark distributions at these photospheric heights and field strengths are extremely small. The fields required to produce these shifts are much greater than the fields which occur with any real degree of probability. The most likely fields are even smaller. (See table 4-4 for expected fields (i.e. normal fields).)

As the quasi-static damping has such a small effect, it can be completely neglected in the solar photosphere. As this estimate of the importance of quasi-static damping neglected Doppler broadening, the effect compared to the total width of the Voigt profile will be even smaller.

Thus, damping should introduce no asymmetry, and, as the damping will be symmetric, unlike the Doppler broadening, strongly damped lines should show less asymmetry than weakly damped lines. This is observed in the solar spectrum (see chapter 1).

#### **4.8: Damping Constants**

The methods described above (such as the Brueckner-O'Mara theory) can be used to determine damping constants (i.e. line widths) for many cases. At other times it may not be possible to accurately determine a damping constant for a particular line, such as when the electronic configuration of the element is such that it proves particularly difficult to appropriately describe the atom at the ranges at which important contributions to the damping are made. Some solar lines are still unidentified, which makes any such calculation impossible.

For some cases where the damping cannot be theoretically calculated, experimental damping constants have been measured. Experimental damping constants for collisions with atomic hydrogen, however, are few in number, and generally not very accurate.

Spectral synthesis of the line profile can be used to determine the damping constant that gives the best fit to the observed line profile. For a determination like this to be accurate, accurate  $gf$ -values are required. The contribution of velocity fields to the line profile must also be well known, or large errors can be introduced into the empirical damping constant.

