Chapter 6: Granulation

6.1: Solar Mass Motions

The photosphere is far from static; it exhibits a wide range of motion, with scales ranging from smaller than can be resolved to comparable with the size of the sun. The velocities associated with the large scale flows are small and will have little effect on spectral lines. The various motions comprising the large scale flows are well separated spatially so unless a spectrum is observed over a large area of the solar surface, only a single portion of the large scale flow will affect the spectrum. The smaller scale motions, with higher velocities, will have more effect and will tend not to have their constituent motions resolved even in spectral observations of fairly small regions of the photosphere. These small scale motions include the solar granulation.

6.1.1: Large Scale Motions

The sun exhibits a wide range of large scale motions, including mesogranulation, supergranulation, giant cells, differential rotation, meridional flows and torsional oscillations.[1] As these motions can be spatially resolved, their vertical velocities can be measured reliably; the velocities are generally quite small, particularly for the very large scale motions (all of the above except supergranulation and mesogranulation). The physical characteristics of such motions can, and have been, investigated and are reasonably well known, even if the forces driving such motions are less well known.

Supergranulation consists of cells of about 30 000 km across and lifetimes of about 1 or 2 days. Typical supergranulation horizontal velocities are about 0.5 kms^{-1},

---

[1] For a review of large scale motions, see Bogart, R. S., “Large-scale Motions on the Sun: an Overview” *Solar Physics* **110**, pg 23-34 (1987).

and vertical velocities are much smaller, about 0.03 kms^{-1}. The mesogranulation pattern is smaller and faster, with cell sizes of about 7000 km and lifetimes of a few hours. Horizontal and vertical r.m.s. velocities of 0.75 kms^{-1} and 0.3 kms^{-1} have been measured.[2]

As these velocities are quite low compared to typical granular velocities, their effects can be neglected, especially at disk centre when the horizontal velocities will have no effect.

6.1.2: Granulation

The solar granulation is the smallest motion resolved on the surface of the sun; it consists of small regions of hot rising material (a granule) surrounded by relatively thin regions of cooler falling material (the intergranular space). A granular cell thus consists of the rising granular centre and the intergranular region in which the material, having risen, falls back into deeper regions.

A granular cell is typically about 1000 km across, and has a lifetime of about 8-10 minutes. Typical vertical and horizontal velocities of about 1 kms^{-1} and 2 kms^{-1} can be seen. From these velocities and typical lifetimes, it is easily seen that the granulation does not consist of steady cyclical flow, but rather of material which rises, falls again, and then reforms into different rising regions as the lifetime of a granule is too small to allow a steady circulation. The overall appearance of the granulation pattern is typical of convective flow; the lack of a steady flow is hardly surprising considering the fact that the photosphere is stable against convection. It is not possible to directly measure the height dependence of the flow velocities, as the emergent radiation does not emerge from a unique height, but rather from a range of heights. It is also difficult to try to predict what the height variation should be; as the convective flow proceeds into the photosphere (which is stable against convection) the mass flow should decrease, but the density also decreases rapidly with increasing height, which would result in increasing flow velocities for a constant mass flow. It is perhaps not

---

[2] See Antia, H. M., and Chitre, S. M., “Discrete Cellular Scales of Solar Convection” *Solar Physics* **145**, pg 227-239 (1993).

surprising that different workers in this area have reached opposite conclusions, with some deciding that the velocities fall with increasing height, and others deciding that they increase.[3]

6.2: Fluid Dynamics of Granulation

The solar granulation exhibits two separate regimes of velocity fields - there is the large scale flow field which is convective in origin[4] and smaller scale turbulent velocity fields driven by the granular flow.

Theoretical treatments of granulation are difficult, as they involve highly turbulent flow (Reynolds numbers such as 10^9 are typical, much greater than critical values of 1600 for the onset of turbulence). Statistical treatments of turbulence, such as Kolmogorov theory[5](and related theories) give good results for extremely turbulent flow but are really only applicable to turbulence after an equilibrium distribution of turbulent velocities is attained. It is evident that an equilibrium small scale turbulent field does not exist as high resolution observations show that small scale turbulent fields are not uniform, with higher turbulent velocities between the granule centre and the intergranular space where the upflow meets the downflow,[6] while equilibrium

---

[3] The results obtained are likely to depend on the photospheric heights that the velocities are measured at. See Kiel, S.L. and Yackovich, F.H. "Photospheric Line Asymmetry and Granular Velocity Models" *Solar Physics* **69**, pg 213-221 (1981) for an example of this. Also, even if velocities are found to decrease in the lower photosphere, at some higher altitude the velocities must increase to the observed higher chromospheric velocities on the order of 10-30 kms^{-1}.

[4] It was originally uncertain whether the granular flow field was driven by convective processes or if it instead was turbulence driven by larger scale velocity fields. It is now well demonstrated that the large scale granular flow field is not purely turbulent, but is instead convectively driven, as shown by numerical simulations of the solar granulation (see section 6.4.2).

[5] Kolmogorov, A.N. "Local Structure of Turbulence in an Incompressible Viscous Fluid at Very Large Reynolds Numbers" pg 312-318 in "Selected Works of A.N. Kolmogorov. Volume I: Mathematics and Mechanics" Kluwer Academic Publishers, Dordrecht (1991).

[6] Nesis, A., Hanslmeier, A., Hammer, R., Komm, R., Mattig, W. and Staiger, J. "Dynamics of the Solar Granulation II. A Quantitative Approach" *Astronomy and Astrophysics* **279**, pg 599-609 (1993).

turbulence theories predict that the small scale turbulent velocities should be indistinguishable spatially and temporally.

6.2.1: Photospheric Viscosity

The viscosity of a gas results from the diffusion of particles between streamlines, with consequent momentum transfer. The viscosity coefficient will be given in terms of the particle density, the mean momentum of a particle and the mean free path l by

$$\eta = 2aNm\bar{v}l \qquad (6\text{-}1)$$

where a is a constant of proportionality. The mean free path depends on the particle density and a collision cross-section σ

$$l = \frac{1}{\sqrt{2}\sigma N} \qquad (6\text{-}2)$$

so the viscosity coefficient can be written as a function of temperature

$$\eta = \frac{K}{\sigma\sqrt{m}}\sqrt{T} \qquad (6\text{-}3)$$

where K is a constant. As the photosphere is composed of many types of particles, they will all need to be considered to find the total viscosity,[7] by finding a suitably averaged viscosity, with the contribution of a particle species being proportional to the species viscosity and the fractional abundance of the species. We can note that the dominant source of viscosity in the photosphere will be atomic hydrogen, due to its high abundance, with the next largest contributions being due to helium (about 0.05 of the hydrogen contribution). In a more highly ionised atmosphere, the viscosity due to electrons can be important, and can dominate the viscosity, due to the low mass and consequently relatively high viscosity of electrons.

The viscosity will vary slowly throughout the photosphere as it is a function of temperature (and the degree of ionisation) and is independent of pressure. Photospheric viscosities can be expected to be on the order of 4-5 $\times$ 10^{-4} poises (dyn-

---

[7] See pg 597 in Anderson, J.D. “Hypersonic and High Temperature Gas Dynamics” McGraw-Hill (1989).

sec cm^{-2}). This viscosity, which is not much greater than terrestrial gas viscosities, will only affect very small motions (with high velocity gradients as a result of their small size) significantly.

The smallest scale turbulent motions will be dissipated by the viscosity, and new small scale motions will be created by the cascade of kinetic energy from larger scale turbulent elements. In this way, the larger scale motions will be affected by the viscosity. If an equilibrium distribution of turbulent kinetic energies exists, an effective pseudo-viscosity can be found in terms of larger scale turbulent motions, thus bypassing the need to treat the smaller scale motions in detail. Such a treatment of turbulent pseudo-viscosity (also known as eddy viscosity) will be less accurate if an equilibrium distribution of kinetic energies does not exist.

6.2.2: Turbulence at Very High Reynolds Numbers

At **sufficiently** high Reynolds numbers, and at a sufficient distance from boundaries, the turbulent field should be isotropic and spatially homogeneous once an equilibrium distribution of velocities has been attained. Under such conditions, useful results can be obtained from statistical turbulence theories. The solar photosphere unfortunately does not fit within these conditions, so such theories are only of limited value. The large scale flow fields cannot be predicted from statistical theories as the boundary conditions have too great an effect on the flow (apart from the discrepancy between the observed flow pattern and statistical predictions of pure turbulence, this is readily shown in numerical simulation of granular flow).

Some useful results from statistical theories can be obtained for the small scale velocity fields, which are purely turbulent in origin. Namely, they can be expected to be isotropic and to have nearly Gaussian velocity distributions.[8] The magnitude of the small scale field will vary with position, as the small scale fields in the photosphere cannot reach a uniform distribution due to the effects of stratification (see section 6.2.2 below). The smallest turbulent elements that can exist will be of a size where their energy is lost due to viscosity, rather than to smaller turbulent elements. Turbulent

---

[8] These are standard results of various statistical turbulence theories.

motions smaller than this limiting size will be damped by viscosity and if there are no small turbulent elements, larger turbulent elements will give rise to progressively smaller elements until the limiting size is reached unless another mechanism interferes with this process.

Photospheric conditions cannot be duplicated in the laboratory experiments in fluid flow and convection due to the low densities, compressibility, high Reynolds numbers and the strong stratification. Laboratory results therefore cannot be directly applied to the photosphere.

6.2.3: Turbulent Flow in a Highly Stratified Medium

The stratification of the photosphere has important effects on the flow therein. There will be effects on both the large scale granular flow and on the small scale turbulent velocities.

In the granular flow, the total mass flow must be conserved. As an upwards moving element rises, its density drops rapidly. As the density falls, either the volume flow must increase to maintain the same mass flow, or the mass flow must fall. In the first case, either the element must expand horizontally to increase the volume, or the flow velocity must increase. In the second case, there must be a corresponding outflow of mass from the element to ensure conservation. Similarly, a downflow will either gain mass or slow down, or be laterally compressed. In the photosphere, as a granular centre rises, it cannot readily expand horizontally, as the surface of the photosphere is densely packed with granules, so the granular centre can be expected to either increase its upwards velocity or lose mass to the downflow. If an upwards moving element is considered, the maximum horizontal flow velocity (which will occur along the border) can be determined from the change in the mass flow rate due to the rising element. For a rising element with cross-sectional area A and a corresponding mass flow rate of $\rho A V_V$, where V_V is the speed of the upflow, there will be a horizontal mass outflow of $\rho L dh V_H$, where L is the length of the border of the flow along which the horizontal outflow occurs and dh is the vertical distance involved. The horizontal flow speed is given by

$$\begin{aligned} V_H &= \frac{-A}{\rho(h)L}\frac{d\left(\rho V_V\right)}{dh} \\ &= \frac{-A}{L}\left(\frac{dV_V}{dh}+\frac{V_V}{\rho}\frac{d\rho}{dh}\right). \end{aligned} \qquad (6\text{-}4)$$

For downflows, the relationship is exactly the same, where the inflow speed is determined by the rate of change of the downflow speed. Thus, in the region where the upflow stops, high horizontal velocities can be expected.

A turbulent element in an upflow will expand. This will tend to reduce the population of small turbulent elements, and if the upflow is sufficiently rapid, this depopulation of small turbulent elements by expansion will exceed their creation by larger turbulent elements. This process will stop an equilibrium distribution of turbulent velocities from being attained. Thus, the turbulence in upflows is expected to be weak.

The reverse of this process will occur in downflows, with turbulent elements being compressed. If an element is compressed to a size where it will be strongly affected by viscosity, its turbulent kinetic energy will be rapidly dissipated. This destruction of overly small turbulent elements will prevent the total turbulent energy from increasing indefinitely with compression, but the downflows can still be expected to have significantly higher turbulent velocities than the upflows.

Thus, spectral lines formed in downflows should show greater turbulent widths than those observed in upflows. This prediction is confirmed by observations.[9]

---

[9] See, for example, Kiselman, D. “High-Spatial-Resolution Solar Observations of Spectral Lines Used for Abundance Analysis” *Astronomy and Astrophysics Supplement Series* **104**, pg 23-77 (1994).

6.3: The Structure of Granulation

6.3.1: Structure of a Granular Cell

There are a number of problems with attempting to directly determine the structure of a granular cell. As the cell is typically only about 1000 km across, only the larger features within the cell can be resolved. Thus, while direct measurements of some properties of granulation can be attempted, many elements of granular structure will have to be determined indirectly. Such indirect determinations can consist of using the solar spectrum, particularly the shapes of spectral lines, to probe granular structure, or attempts to theoretically predict granular structure.

The elements of the structure of a granular cell are the variations in temperature, pressure and density throughout the cell, the physical dimensions of the granular cell, the large scale velocity field (the granular flow field) and the small scale turbulent velocity field.

6.3.2: Temperature Variation within a Granular Cell

At the depths where the granulation is driven, the rising granular centre must be hotter than the falling material in the intergranular space. In the photosphere, this will not necessarily be the case, as the photosphere is stable against convection, and such motions must be caused by deeper convective motions overshooting into the (stable) photosphere. As a granular centre rises, it will cool through expansion, and due to the superadiabatic temperature gradient in the photosphere, this cooling can be quite rapid. There will also be efficient radiative cooling. Thus, we can expect that deep in the photosphere, there will be large temperature differences between the upflows and downflows, and higher in the photosphere, these temperature differences will be smaller.

That there is a temperature difference in the lower photosphere is readily seen from the continuum intensity. The granular centres are brighter in the continuum than the intergranular space (this brightness difference is responsible for the granulation being visible).

Roudier and Muller[10] measured intensity variations in wavelengths 5720 Å to 5780 Å, with a spatial resolution of 0″.25, and found the r.m.s. intensity variation to be 8.1%. This corresponds to an r.m.s temperature variation of 2%, or 130°K at $\tau_{5000} = 1$. Greater temperature differences must exist at greater depths, but the temperature differences higher in the photosphere should be no greater than this, and could be much smaller. From high spatial resolution observations given by Nesis et al.[11] the brightest regions at 4912.5 Å are 7.4% brighter than the mean intensity at this wavelength, and the dimmest regions are 9.5% fainter. Kiselman[12] observed bright regions at 6153 Å which were 22% brighter than the mean intensity and dark regions which were 12% fainter. This large intensity difference for the bright region corresponds to a temperature difference of 370°K.

It has been shown that the temperature variations rapidly become smaller as the height increases.[13] Thus, the temperature variation will only need to be considered for lines forming deep in the photosphere. Lines forming higher in the photosphere will be formed in regions of uniform temperature (with respect to horizontal variations).

Lastly, it can be noted that the temperature variation in the lower photosphere will affect spectral lines since the increased continuum intensity from hotter regions will result in a greater contribution to the total line profile from the upflows than would be expected from the area occupied by upflows would indicate. The contribution from downflows will be correspondingly smaller.

---

[10] Roudier, Th. and Muller, R. “Structure of the Solar Granulation” *Solar Physics* **107** pg 11-26 (1986).

[11] Nesis, A., Hanslmeier, A., Hammer, R., Komm, R., Mattig, W. and Staiger, J. “Dynamics of the Solar Granulation II. A Quantitative Approach” *Astronomy and Astrophysics* **279**, pg 599-609 (1993).

[12] Kiselman, D. “High-Spatial-Resolution Solar Observations of Spectral Lines Used for Abundance Analysis” *Astronomy and Astrophysics Supplement Series* **104**, pg 23-77 (1994).

[13] See, for example, Hanslmeier, A., Mattig, W. and Nesis, A. “High Spatial Resolution Observations of Some Solar Photospheric Line Profiles” *Astronomy and Astrophysics* **238**, pg 354-362 (1990).

6.3.3: Pressure Variation within a Granular Cell

Pressure variations must exist within the granulation. As there must be horizontal flows (as the vertical velocities do not increase exponentially with height and decreasing density), the pressure differences needed to drive these horizontal flows must exist.

The horizontal pressure variations will not be as important as the temperature fluctuations. This can be seen from the dynamics of the granular flow[14] and from observations of line profiles. The damping wings of spectral lines will be the portion of the line profile most affected by pressure fluctuations, and observations show fluctuations in the strengths of wings of lines. Such wing strength fluctuations were investigated by Kneer and Nolte[15] who concluded that the effects of temperature variations are much greater (by a factor of 5 or 6) than the effects of pressure variations. This makes it difficult to directly measure pressure variations.

If the temperature fluctuations were well known, their effects could be separated from those of pressure fluctuations, which could then be directly measured. This would require a better knowledge of temperature variations than is currently available. The pressure can also be determined hydrodynamically if the granular flow and temperature are known, without recourse to further observations.

6.3.4: Density Variation within a Granular Cell

Horizontal variations in density will be related to variations in pressure and temperature. The effects of density variations will be the combination of the effects of the matching temperature and pressure variations, and the density variations do not need to be considered separately.

---

[14] Nordlund, Å. "Numerical Simulations of the Solar Granulation I. Basic Equations and Methods" *Astronomy and Astrophysics* **107**, pg 1-10 (1982).

[15] Kneer, F. and Nolte, U. "On the Fluctuation of Wing Strengths as Diagnostics of the Solar Atmosphere" *Astronomy and Astrophysics* **286**, pg 309-313 (1994).

6.3.5: The Horizontal Variation of the Granular Flow

The granular flow velocity varies strongly with horizontal position; the upflows move upwards and the downflows move downwards. The downflows, occupying a smaller fraction of the solar surface than the upflows, have correspondingly higher flow speeds. The speed of the upflow and downflow, or at least the differences between them, can be approximately measured from the red- and blue-shifts of spectral lines observed with high spatial resolution. This only gives an approximate result as the spectral lines do not form at single heights, but rather over a range of heights, so only an average velocity weighted by the contributions to the line from different heights can be found in this manner.

Using Doppler shifts such as these, only the line-of-sight velocities can be found, which, at disk centre, will be the vertical (upwards and downwards) flow velocities. The horizontal flow speeds can, however, be found from the vertical flow using mass flow conservation.

As upflows occupy a larger fraction of the surface than downflows, and the continuum formed at the base of upflow (where the temperature is higher) is brighter, the major contribution to the average line profile is from upflows. The average line profile should then be blue-shifted (with a shift due mostly to the upflow speed). To accurately measure the blue-shifts of spectral lines, accurate line profiles and accurate laboratory wavelengths are needed. The gravitational redshift of 636 ms^{-1} must also be taken into account. Measurements of solar line shifts by Dravins et al.[16] show that strong lines have blue-shifts of 200-300 ms^{-1} while weak lines have blue-shifts of 300-400 ms^{-1}. Wavelengths shifts of spectral lines due to convection should only depend on the depth of formation of the spectral lines, but as laboratory wavelengths are often only accurate to 100 ms^{-1} or worse, the large scatter in their results may be due to the laboratory wavelengths used rather than any variation of the actual shifts of lines in the

---

[16] Dravins, D., Lindegren, L. and Nordlund, Å. "Solar Convection: Influence of Convection on Spectral Line Asymmetries and Wavelength Shifts" *Astronomy and Astrophysics* **96**, pg 345-364 (1981).

photosphere. The results obtained by Dravins et al. agree with earlier observation of solar line shifts.[17]

As the horizontal variation in the vertical mass flow velocity is strongly asymmetric, it can be expected to contribute significantly to spectral line asymmetries near disk centre. The horizontal flow will not affect spectral lines near disk centre, but will affect line profiles near the limb. The horizontal mass flow should be symmetric. If spectral lines are observed near the limb, the vertical flow will have little effect, and the line should not be shifted by the granular flow. Spectral lines observed at the limb exhibit only the expected gravitational redshift[18] which is in agreement with these expectations.

6.3.6: The Vertical Variation of the Granular Flow

As mentioned above in section 6.3.5, the wavelength shift of a spectral line depends on the strength of the line. As the depth of formation of spectral lines depends on their strength, this strongly suggests that the vertical mass flow is not constant with height. Measurement of the variation of the vertical flow with height in the photosphere is difficult, so it is not overly surprising that all possible conclusions have been drawn in previous work - that the flow speed is constant, increases with height, decreases with height, or some combination of these cases.

As a flow moves upwards into a region which is stable against convection, it is expected that the mass flow will rapidly decrease. Whether or not the speed of the flow will increase or decrease is not so clear, due to the strong density stratification of the photosphere. This question is readily resolved from the observations of line shifts - strong lines, forming higher in the photosphere, show smaller wavelength shifts than

---

[17] See pg 346 in Dravins, D., Lindegren, L. and Nordlund, Å. "Solar Convection: Influence of Convection on Spectral Line Asymmetries and Wavelength Shifts" *Astronomy and Astrophysics* **96**, pg 345-364 (1981) for a brief review of earlier wavelength shift measurements.

[18] See pg 347 in Dravins, D., Lindegren, L. and Nordlund, Å. "Solar Convection: Influence of Convection on Spectral Line Asymmetries and Wavelength Shifts" *Astronomy and Astrophysics* **96**,

weaker lines from deeper in the photosphere. From this, it is readily seen that the flow velocity must decrease with increasing height. The decrease is not necessarily great, as the line shifts fall from 300-400 ms^{-1} for weak lines to 200-300 ms^{-1} for strong lines.

6.3.7: The Horizontal Variation of the Turbulent Velocity Field

High spatial resolution spectra show that the small scale turbulent velocity field (the microturbulence) is not uniform horizontally.[19] This is in accordance with our expectations from the behaviour of fluid flow in highly stratified atmospheres. Lines formed in the darker (and red-shifted) intergranular space are significantly broader than lines formed in the rising granular centre. The transition region between the upflow and the downflow, where there is a high velocity gradient, also displays high turbulence.

The difference in the turbulent velocities between upflows and downflows are such that the blue wings of both blue-shifted narrow lines formed in granular centres and red-shifted broader lines formed in the intergranular region roughly coincide (see figure 6-1 below).

---

pg 345-364 (1981) for a brief review of wavelength shift measurements for spectral lines observed at the limb.

[19] See Hanslmeier, A., Mattig, W. and Nesis, A. "High Spatial Resolution Observations of Some Solar Photospheric Line Profiles" *Astronomy and Astrophysics* **238**, pg 354-362 (1990), Hanslmeier, A., Mattig, W. and Nesis, A. "Selected Examples of Bisector and Line Parameter Variation over a Granular-Intergranular Region" *Astronomy and Astrophysics* **251**, pg 669-674 (1993), Nesis, A., Hanslmeier, A., Hammer, R., Komm, R., Mattig, W. and Staiger, J. "Dynamics of the Solar Granulation II. A Quantitative Approach" *Astronomy and Astrophysics* **279**, pg 599-609 (1993) and Kiselman, D. "High-Spatial-Resolution Solar Observations of Spectral Lines Used for Abundance Analysis" *Astronomy and Astrophysics Supplement Series* **104**, pg 23-77 (1994).

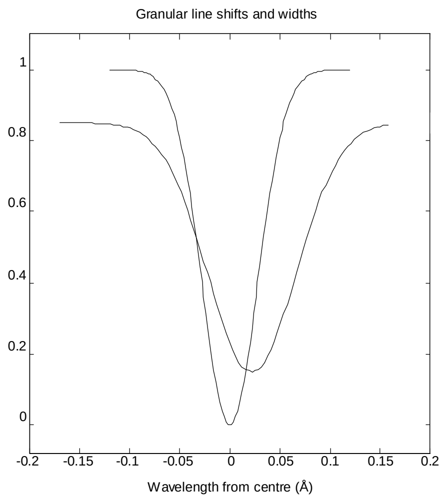

Figure 6-1: Blue Wing Coincidence

As a result of this coincidence of blue wings, the granulation is difficult to see while on the blue side of a line, and easy to see when on the red side.[20]

6.3.8: The Vertical Variation of the Turbulent Velocity Field

The vertical variation of the turbulent velocities is difficult to determine. Due to the expansion of upflows and the compression of downflows, the turbulent velocities are expected to increase with increasing depth. It may prove difficult to extract much information regarding the vertical variation of the microturbulence from the solar spectrum.

It can be noted that standard plane-parallel microturbulence-macroturbulence spectral synthesis is often performed using depth independent microturbulence, as the results obtained are almost identical with those obtained using depth dependent microturbulence. The weak effect of small variations in the microturbulence make them difficult to determine, but it can be seen that, as the effect of the spectrum is

---

[20] See pg 604 in Nesis, A., Hanslmeier, A., Hammer, R., Komm, R., Mattig, W. and Staiger, J. "Dynamics of the Solar Granulation II. A Quantitative Approach" *Astronomy and Astrophysics* **279**, pg 599-609 (1993) for a summary of observations relating to the blue wing coincidence.

small, the variation of microturbulence across the heights where photospheric spectral lines form cannot be overly large.

6.3.9: Granular Velocity Structure

The various aspects of the velocity structure of a granular cell are shown schematically below (see figure 6-2 and figure 6-3).

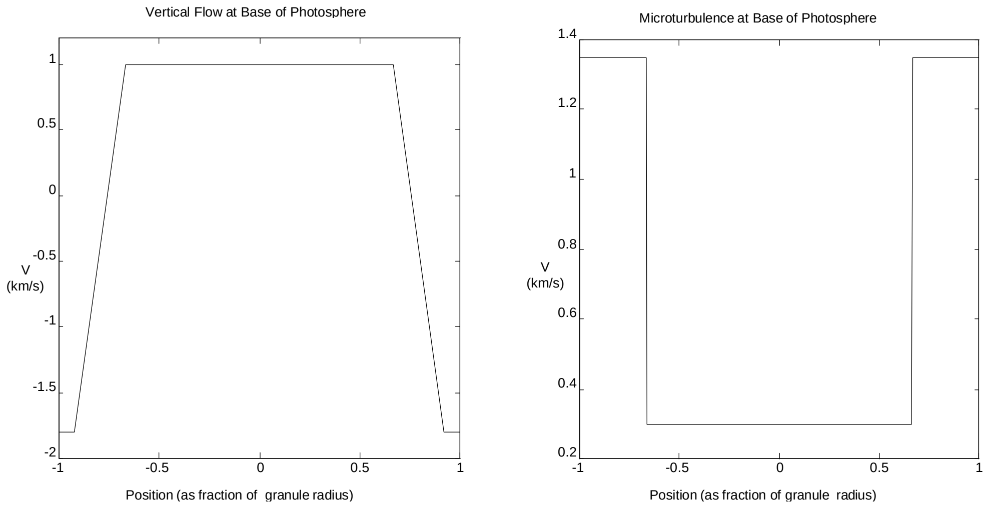

Figure 6-2: Horizontal Variations within a Granular Cell

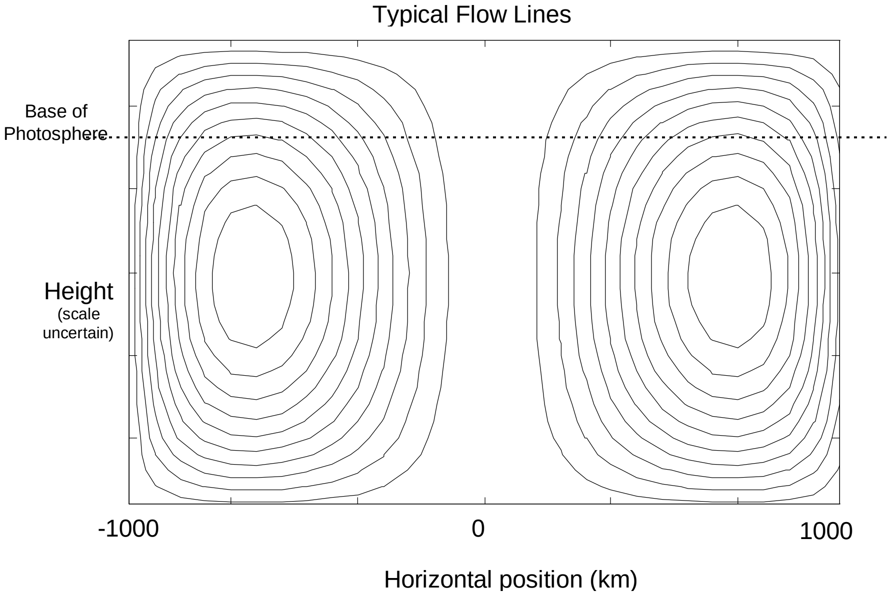

Figure 6-3: Flow within a Granular Cell

Only the upper portion of the flow shown in figure 6-3 is observed. The rest of the flow will be below the observable atmosphere. Flows of this general nature are typical of convective processes. The main features distinguishing the solar convection from other common flows are the extremely high Reynolds numbers (and resultant turbulence) and the strong stratification of the atmosphere.

These granular properties can be used as the basis of a simple granular model. This procedure is discussed in detail in chapter 7.

6.3.10: Variations Between Granules

So far, the properties of a "typical" granule have been considered. As all granules are not the same, the variations between granules will need to be considered. The main consideration will be variations in flow velocities and turbulent velocities. As the properties of individual granules are rarely measured to a high degree of accuracy, the variation between granules is poorly known. In the absence of reliable data, it is liable to prove useful to assume that any variation is Gaussian.

The actual variation is quite complex, as the properties of a granule vary not only between granules, but also within the short lifetime of an individual granule. Granules also vary in shape and size, and so the variations in the velocity fields between granules could be fairly large. Even if the variation is large, it should be smaller than the variations within individual granules - few granular centres are observed to be falling rather than rising.[21]

---

[21] Bright regions are sometimes observed to be slowly falling, and dark regions sometimes rising. Whether this is due to a variation in the vertical flow speeds or in the brightness is not clear. Bright granular centres generally rise, and the darker intergranular regions generally fall, so the variation in vertical flow speed is not greater than the flow speed, and might be significantly smaller.

6.4: Theoretical Models of Granulation

A number of attempts to deal with granulation theoretically have been made. These range from attempts to apply fairly simple theories through to numerical solution of the equations of fluid flow for granules.[22] Attempts to treat granulation as a simple convective process are investigated, and numerical simulations are examined in detail. These simulations replicate most of the gross behaviour of granules, and provide a firm base for the convective origins of granulation. The simulations show that the granulation behaves in a manner similar to that in which we are led to believe from observations. Both the observations and such simulations can therefore be assumed to be reasonably accurate.

6.4.1: Granules as Rising Spherical Thermals

Granulation is a convective process, and the older treatments of convection, such as the standard mixing length theory, are simple approximations tenuously based on reality. The standard mixing length theory assumes that convection involves elements of the photosphere which, having excess energy, rise for some distance (the Prandtl mixing length) and then dissipates delivering the energy to the surroundings.

An improvement on this approach was used by Ulrich[23] who considered a more detailed model of the rising element. Ulrich modelled the rising element as spherical thermal using the Hill vortex model. This gives a reasonable description of convection in the sun which is similar to models of convection in the terrestrial atmosphere. This then leads to the question of how the granulation is related to convective energy transport in the convection zone. If visible granules can be identified as horizontal cross-sections through such spherical thermals, this description of convection should also successfully model granulation.

---

[22] Nordlund, Å. “Numerical Simulations of the Solar Granulation I. Basic Equations and Methods” *Astronomy and Astrophysics* **107**, pg 1-10 (1982).

[23] Ulrich, R.K. “Convective Energy Transport in Stellar Atmospheres” *Astrophysics and Space Science* **7**, pg 71-86 (1970)

Unfortunately, this simple model does not account for all observable features of the granulation. Observations of granules growing larger and splitting into two new granules and two granules merging into one are not explained by this model. The model also predicts that granules should appear circular. While most granules are roughly circular, odd-shaped granules are also reasonably common. Since there is a minimum size to rising spherical thermals, the model also predicts the existence of too few small granules.

The spherical thermal model is therefore not an adequate model for granulation. As granulation is poorly modelled by such a model of convective energy transport, it will be necessary to approach the problem in more detail. The hydrodynamics of highly turbulent flow in a highly stratified medium will need to be considered. The highly turbulent convection comprising the granulation is liable to behave quite differently from convection associated with much lower turbulence.

6.4.2: Numerical Simulation of Granules

Given a sufficiently powerful computer, it should be possible to numerically solve the equations governing the fluid flow in the photosphere, and thus simulate the behaviour of the solar granulation. Although the sufficiently powerful computer which could be used to treat the problem fully does not yet exist,[24] suitable approximate simulations have been performed by Nordlund.[25] Using a two-dimensional Fourier series representation of the horizontal fluctuations giving rise to the granulation in order to deal with the effectively infinite horizontal extent of the photosphere, and a cubic spline vertical representation, Nordlund obtains a suitable grid of points for dealing with the problem. If the equations of motion for the photosphere can be solved

---

[24] The difficulty of completely solving the problem can be readily seen. The viscosity is only effective on lengths of the order of 1 cm, while the photosphere alone (ignoring the deeper regions which are responsible for the formation of the granulation) is several hundreds of kilometres thick. A large horizontal extent will also need to be considered. The full model atmospheres problem (a major undertaking in its own right) involves dealing with a very large number of spectral lines.

for reasonably closely spaced points in time, a picture of the evolution of granulation should be obtained.

These simulations reproduce the overall appearance of the solar granulation. Rising granular centres surrounded by a more rapidly falling intergranular space are formed. The granules tend to grow in horizontal extent with time until they break up into smaller ones. Exploding granules also form in the simulations.

Such simulations, although they reproduce the observed granulation well, are not well suited for determining space and time averaged spectra. The simulation, at any time, only gives an instantaneous picture of the granulation, from which an instantaneous spectrum can be obtained. The emergent spectrum would have to be calculated for a large number of horizontal positions within the simulation to obtain a spatially-averaged spectrum. This would have to be performed over many time steps to obtain a temporal average. The entire process would need to be repeated in order to obtain a spectrum using different line parameters.

Due to a limited number of grid points available, the simulation cannot directly deal with small scale motions. Ideally, the simulation would extend to a sufficiently small scale so that the viscosity of the fluid directly affects the flow. Nordlund estimates the effect of the smaller scale terms by treating them as an effective viscosity assuming an equilibrium distribution of small-scale motions. As the small scale velocity field does not necessarily reach its equilibrium kinetic energy distribution, this treatment of small scale motions could be the greatest source of error in these simulations. Errors in such viscous effects could lead to incorrect boundary conditions being needed for the simulation results to match the observed behaviour of the granulation. Unfortunately, this simplified treatment of the small scale motions is unavoidable; it is what makes the problem computable at all.[26]

---

[25] Nordlund, Å. “Numerical Simulations of the Solar Granulation I. Basic Equations and Methods” *Astronomy and Astrophysics* **107**, pg 1-10 (1982).

[26] It would be difficult to find a less computable problem in fluid dynamics. The viscosity and the length scale for which viscosity is important is simply too small, and photosphere too large for the entire small scale through to large scale components of the system to be included directly in the computation.

Simulations such as these are perhaps the best window on understanding the basic processes involved in granulation. The small scale phenomena cannot be directly observed due to limits on resolution of observations, and the flows are sufficiently complex to defy exact solutions. To apply such simulations to the problem of spectral line formation, high resolution (in space, time and frequency) spectra are needed. If the spatial resolution is insufficient, the spectra will be averages across relatively large areas of the photosphere, and if the time resolution is insufficient, it will not be possible to observe the evolution of the flow or to obtain suitably stable time averages of spectra. Such a spectral movie should also cover a large area and a long time. Then there would be the problem of time, with granular simulations currently taking the equivalent of several CRAY-1 computer hours.

6.4.3: Recent Developments in Numerical Simulations

The results of various granulation simulations have been compared with each other and with the observed properties of the solar granulation. Gadun and Vorob'yov[27] show that reasonable results can be obtained using two-dimensional simulations, which allow a denser grid of points to be used. They also show that the results obtained depend on the treatment of radiative transfer in the atmosphere, which is not unexpected, given the dominance of radiative processes in the photosphere.

Brummell *et al.* have recently reviewed numerical simulations of solar convection, including large scale motions.[28] Improvements in both the algorithms used and in the size of grids used have given improved results. The largest grids used in calculations are now about 512^3 or 1024^3 points in size, thus covering three orders of magnitude spatially. This, while still substantially smaller than the six (or more) orders of magnitude desired, allows a wide range of motions to be represented.

---

[27] Gadun, A.S. and Vorob'yov, Yu.Yu. "Artificial Granules in 2-D Solar Models" *Solar Physics* **159**, pg 45-51 (1995).

[28] Brummell, N., Cattaneo, F. and Toomre, J. "Turbulent Dynamics in the Solar Convection Zone" *Science* **269**, pg 1370-1379 (1995).

Better treatment of the smaller scales of motions has been obtained through the use of the piecewise-parabolic method to solve the Euler equations for the flow (at the cost of a substantial increase in required computation, as compared with the spectral techniques used by, for example, Nordlund). As a result, motions unresolvable in solar observations can be calculated. This presents obvious difficulties when it comes to comparing the theoretical results with observations. Some properties of these small-scale motions can be measured from the solar spectrum (microturbulence), but their spatial structure cannot.

Although further improvements are required before the full dynamic range of motions can be properly treated, numerical simulations are currently yielding useful results, and can give theoretical predictions for motions inaccessible to observations (either through inadequate resolution or the depth of the motion). Numerical simulations are a powerful tool to study motions in the photosphere, but the range of motions available in the simulations must be extended to the smallest actually occurring scales of motion before full confidence can be placed in the quantitative results of such simulations. The results for the larger scale motions, which are less affected by the small-scale motions and any errors in their calculation, are more reliable, both quantitatively and qualitatively.