Frequency and Temporal Effects in Linear Optical Quantum Computing

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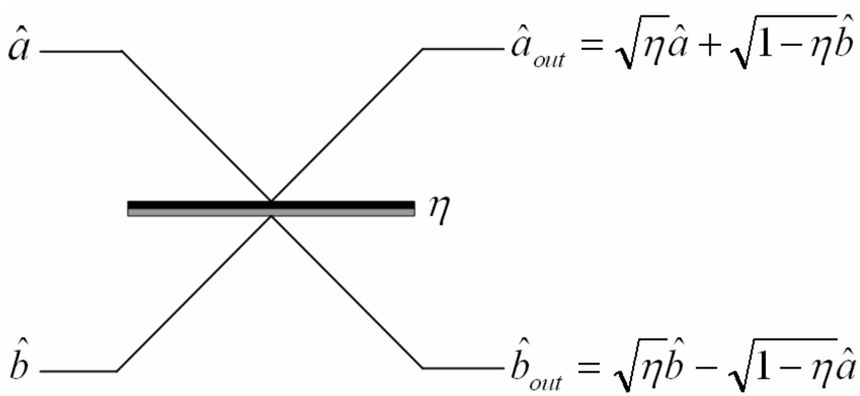
Motivation

- Present LOQC models are highly idealized
- Develop model for ‘realistic’ LOQC circuit behavior
  - Generation of photons
  - Mode-matching problems
  - Non-ideal detectors

The beamsplitter (ideal case)



- For 50/50 Beamsplitter

$$|\psi_{in}\rangle = |1\rangle_a |1\rangle_b$$

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_a |0\rangle_b - |0\rangle_a |2\rangle_b\right)$$

Overview

- Replace ideal state representations with frequency/temporally distributed ones
- Propagate states through circuit
- Examine effects of distribution parameters

The beamsplitter (non-ideal case)

$$|\psi\rangle_{ideal} = |1\rangle_a |1\rangle_b \rightarrow$$

$$|\psi\rangle_{non-ideal} = \left(\int \alpha_\omega |1\rangle_\omega d\omega\right)_a \left(\int \beta_{\omega'} |1\rangle_{\omega'} d\omega'\right)_b$$

$$= \left(\int \alpha_\omega \hat{a}^\dagger{}_\omega d\omega\right)_a \left(\int \beta_{\omega'} \hat{b}^\dagger{}_{\omega'} d\omega'\right)_b |00\rangle$$

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$$|\psi\rangle_{out} = \tfrac{1}{2}\int\int \alpha_\omega \beta_{\omega'} \left(|1\rangle_\omega |1\rangle_{\omega'}\right)_a |0\rangle_b \, d\omega d\omega'$$

$$-\tfrac{1}{2}\int\int \alpha_\omega \beta_{\omega'} |1\rangle_{\omega' a} |1\rangle_{\omega b} \, d\omega d\omega'$$

$$+\tfrac{1}{2}\int\int \alpha_{\omega'} \beta_\omega |1\rangle_{\omega' a} |1\rangle_{\omega b} \, d\omega d\omega'$$

$$-\tfrac{1}{2}\int\int \alpha_\omega \beta_{\omega'} |0\rangle_a \left(|1\rangle_\omega |1\rangle_{\omega'}\right)_b \, d\omega d\omega'$$

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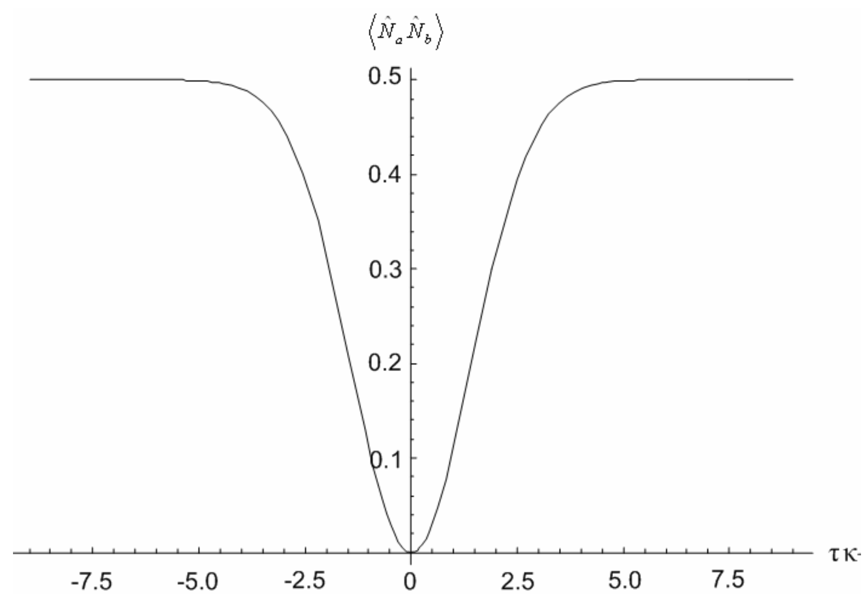
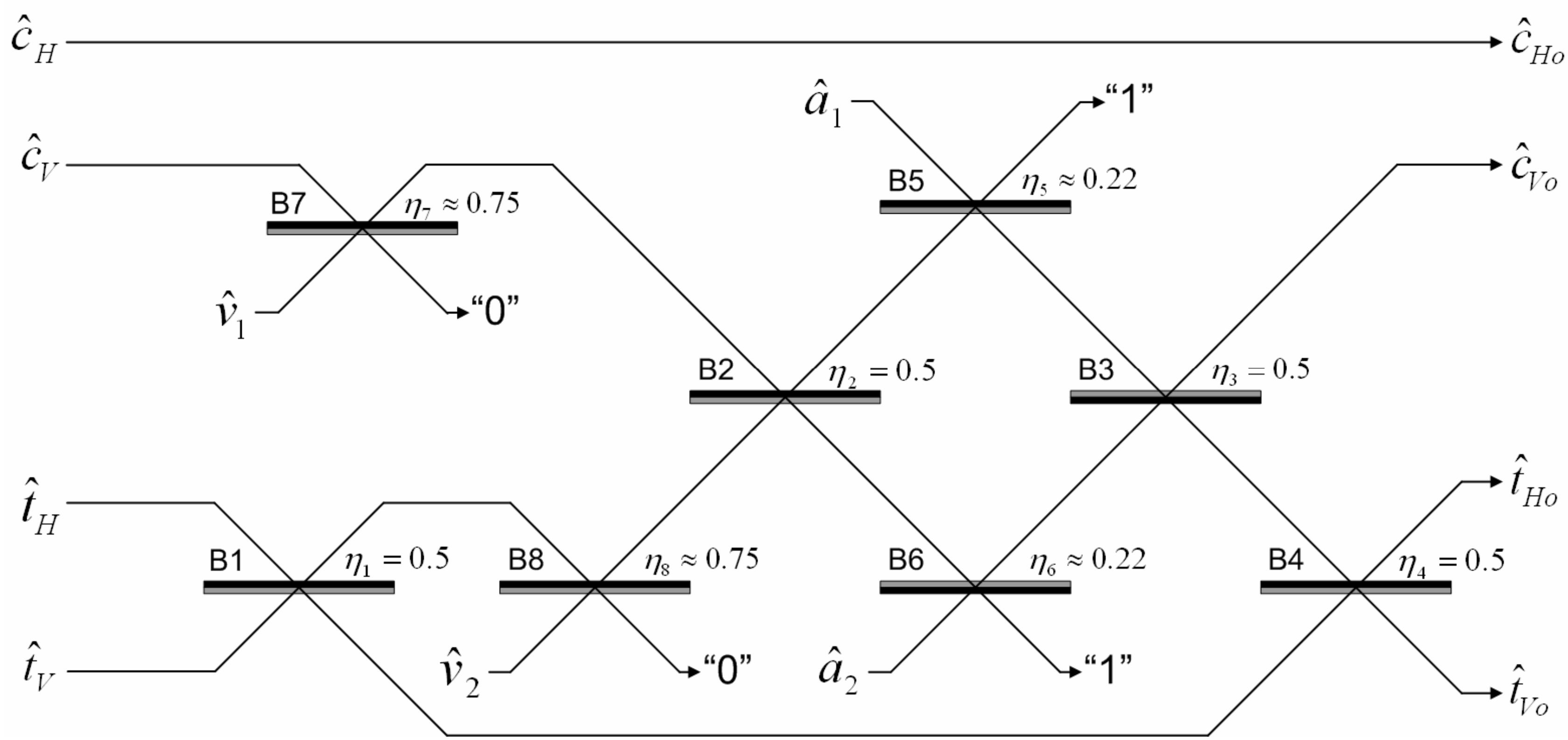


The simplified CNOT-gate



- Dual-rail logic – qubits encoded across two spatial modes
  - polarization encoding → spatial encoding using polarizing beamsplitters
- Non-deterministic – conditional upon measurements at "0" and "1"
  - $P_{success} \approx 5\%$
- Heralded

CNOT-gate inputs

- Ideal case

$$|\psi\rangle_{in} = \left(\alpha\, \hat{c}_H{}^{\dagger} + \beta\, \hat{c}_V{}^{\dagger}\right)\left(\gamma\, \hat{t}_H{}^{\dagger} + \delta\, \hat{t}_V{}^{\dagger}\right)\hat{a}_{1,\omega}{}^{\dagger}\hat{a}_{2,\omega}{}^{\dagger}\,|0\ldots 0\rangle$$

CNOT-gate inputs

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- Non-ideal case

$$|\psi\rangle_{in} = \left(\alpha\int\kappa_\omega\hat{c}_{H,\omega}{}^{\dagger}d\omega + \beta\int\kappa_\omega\hat{c}_{V,\omega}{}^{\dagger}d\omega\right)\left(\gamma\int\sigma_\omega\hat{t}_{H,\omega}{}^{\dagger}d\omega + \delta\int\sigma_\omega\hat{t}_{V,\omega}{}^{\dagger}d\omega\right)$$
$$\left(\int\varepsilon_\omega\hat{a}_{1,\omega}{}^{\dagger}d\omega\right)\left(\int\varepsilon_\omega\hat{a}_{2,\omega}{}^{\dagger}d\omega\right)|0\ldots0\rangle$$

Characterizing CNOT-gate operation

- Fidelity
  - How close is the gate's output to the expected output

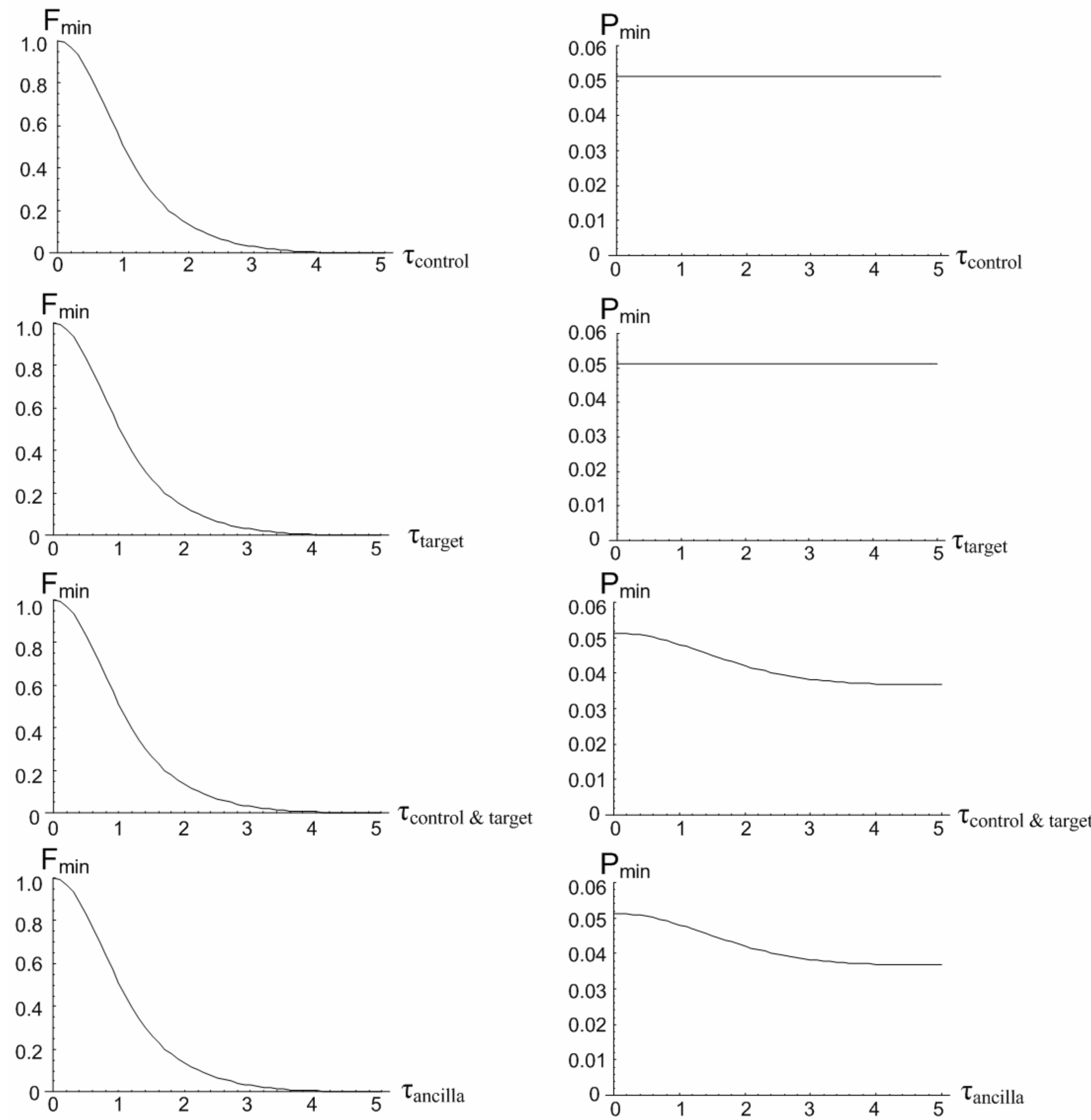
$$F = \frac{\left\langle \psi_{exp} \middle| \hat{\rho}_{out} \middle| \psi_{exp} \right\rangle}{\mathrm{tr}\left(\hat{\rho}_{out}\right)}$$

- Success probability
  - How often does the post-selection process succeed

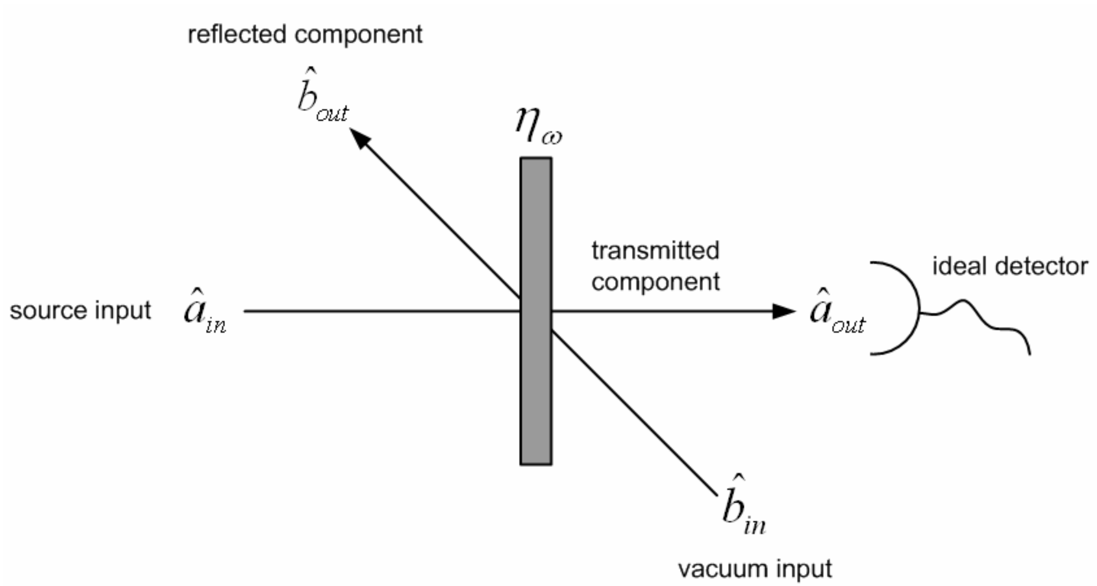
$$P = \mathrm{tr}\left(\hat{\rho}_{out}\right)$$

Effects of time-displaced inputs

- Assume ideal detectors
- All inputs are normal Gaussian distributed (i.e. bandwidth is fixed)
- Time-shifts are introduced between the inputs ($\tau_{control}$, τ_{target} and $\tau_{ancillary}$)
- F_{min} and P_{min} are worst-case results, across all superpositions

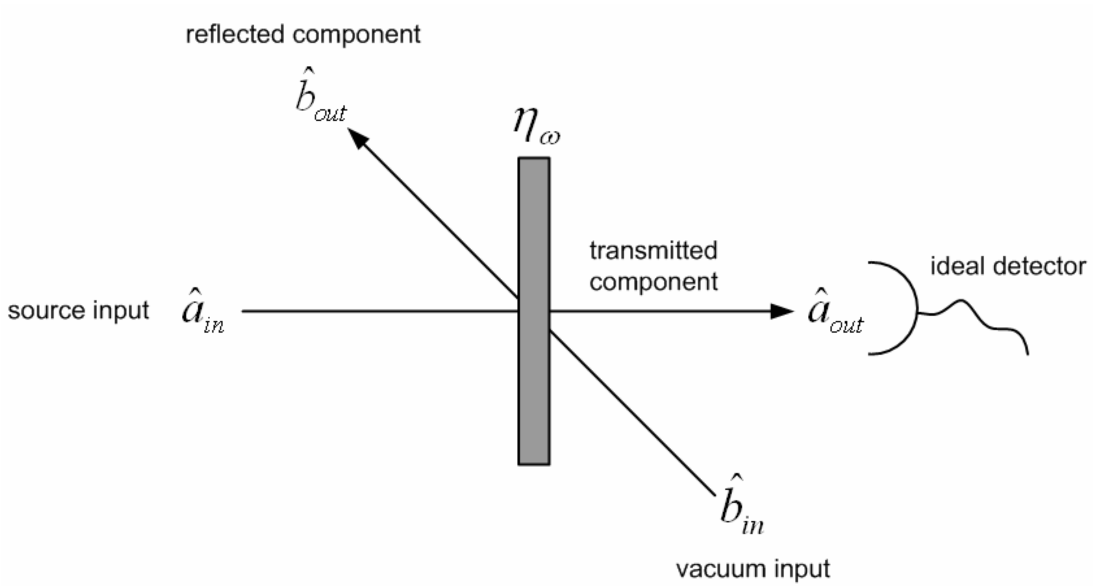


Detector effects - bandwidth



- Model frequency selectivity as beamsplitter with frequency dependent reflectivity
- Followed by ideal detector
- Gaussian frequency response

Detector effects - bandwidth



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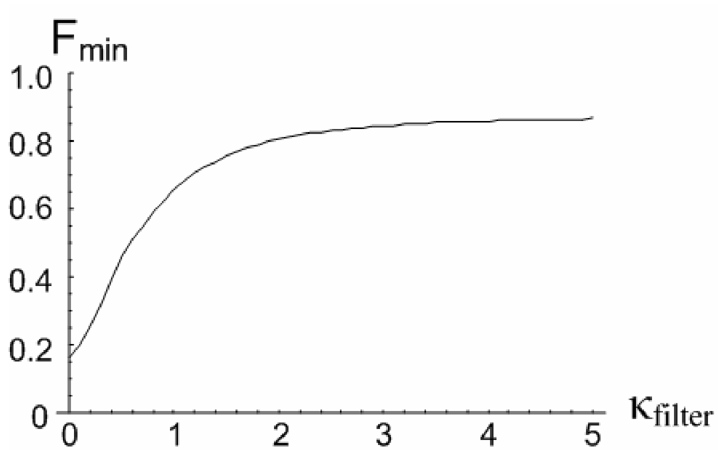
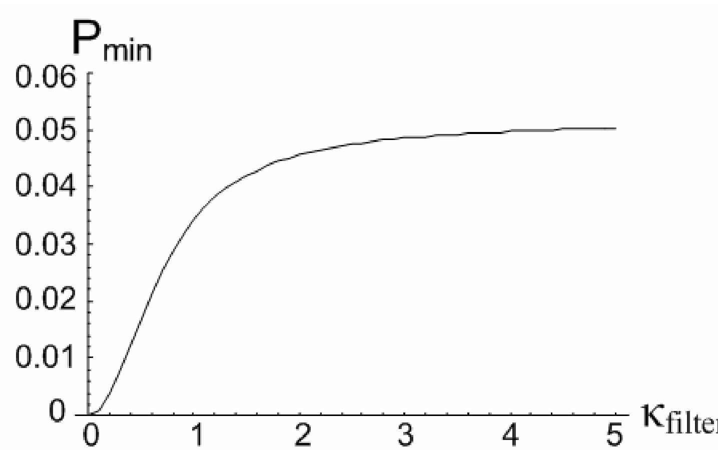
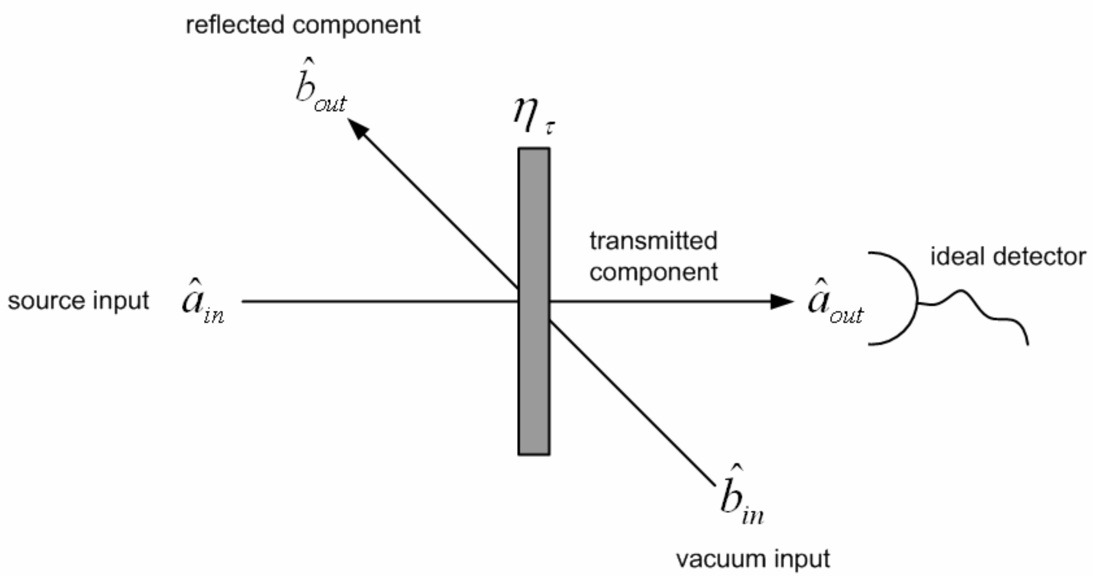
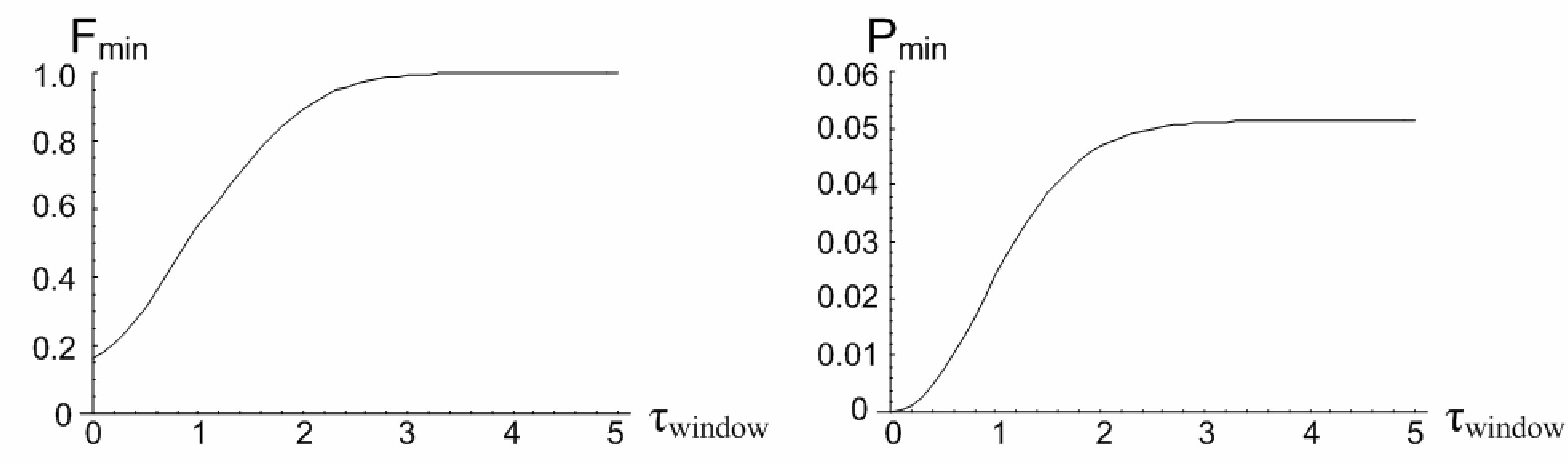




Detector effects – time resolution

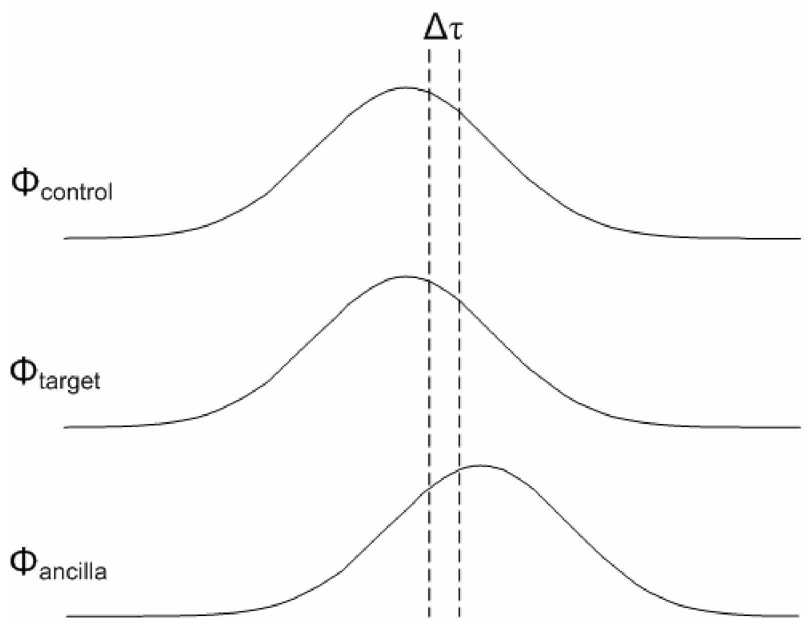


- Model time selectivity as beamsplitter with time dependent reflectivity
- Followed by ideal detector
- Rectangular time response



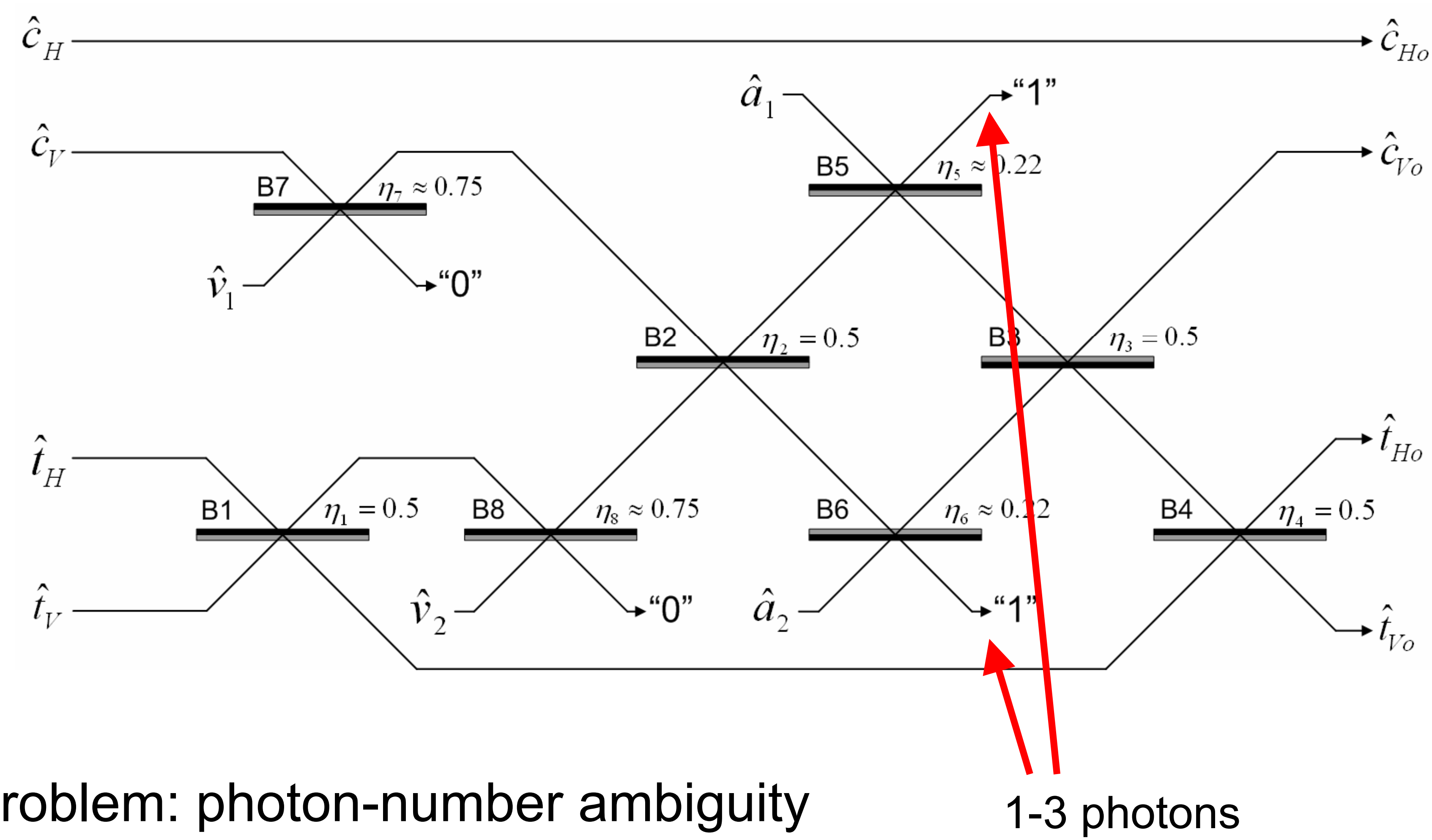
Detector effects – time resolution

- Can we improve fidelity by manipulating the time window of the detectors?
- Similar to HOM-like experiment performed by Legero et. al.



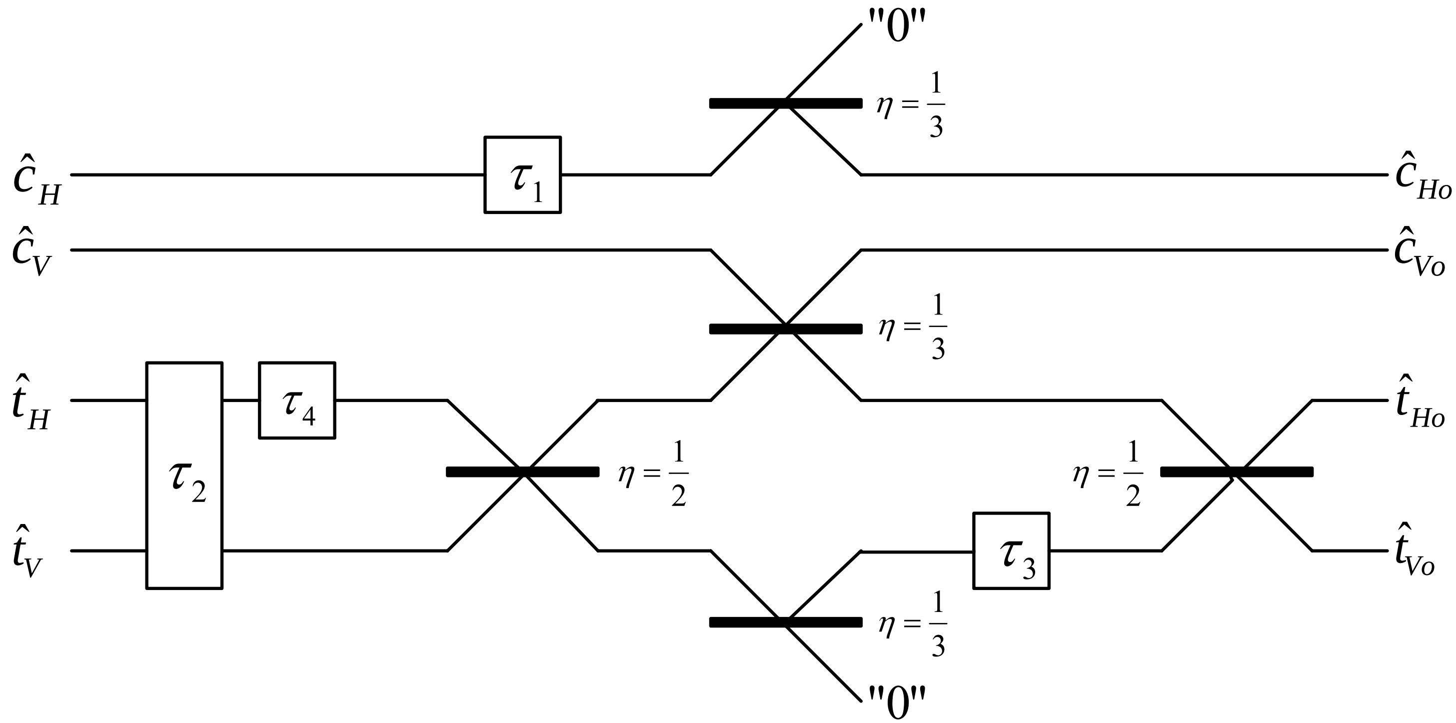
$\Delta\tau$ is region of maximal wave-packet overlap

Detector effects – time resolution



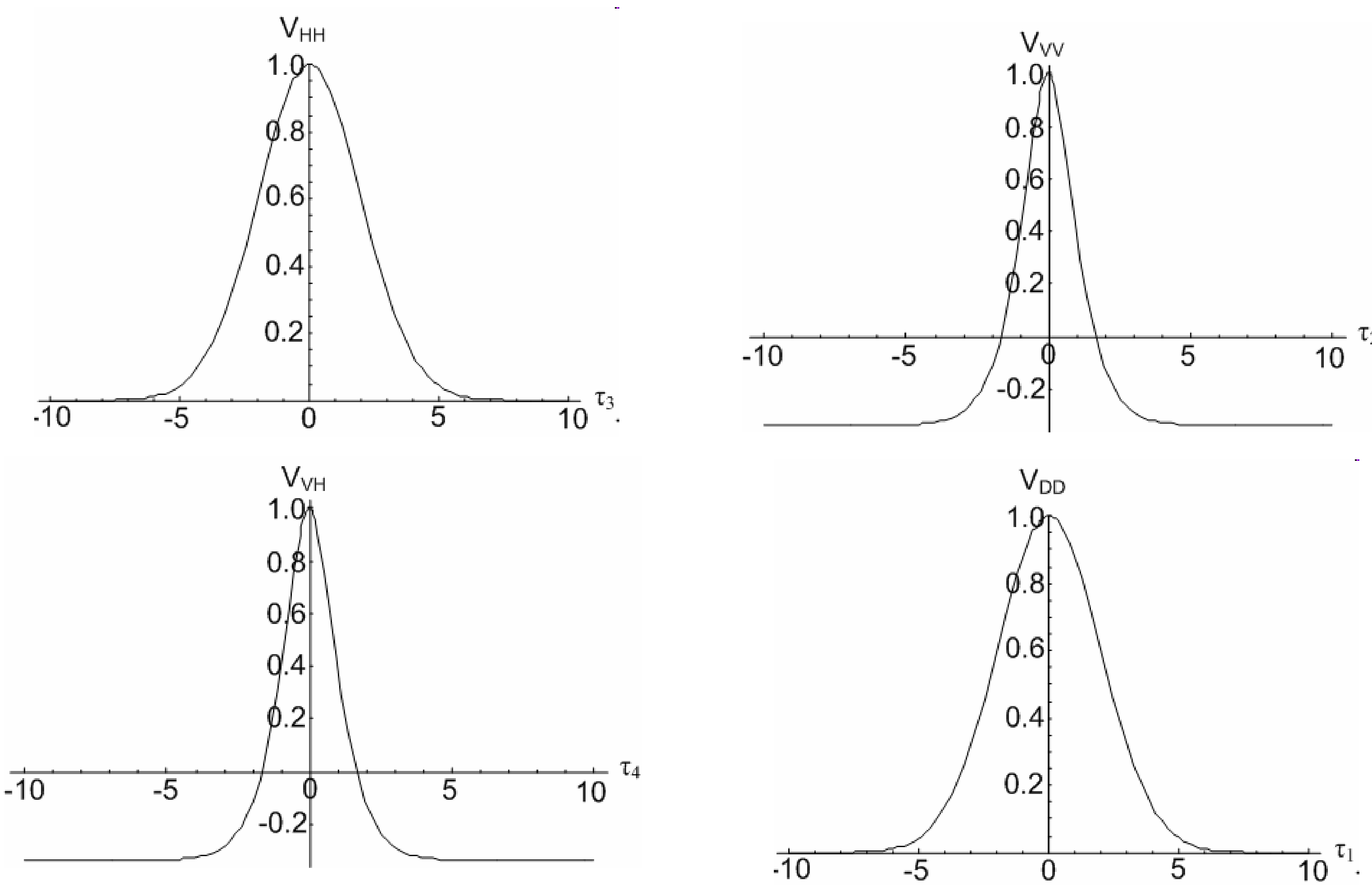
Problem: photon-number ambiguity

Mode-matching effects



- Different implementation of the CNOT-gate
  - Dual-rail logic
  - Operates in the coincidence basis
  - Non-deterministic, $P_{success}$ = 1/9
  - Not heralded
  - Equivalent to the experimental implementation made here at UQ
- τ_1-τ_4 are mode-matching parameters

Mode-matching effects

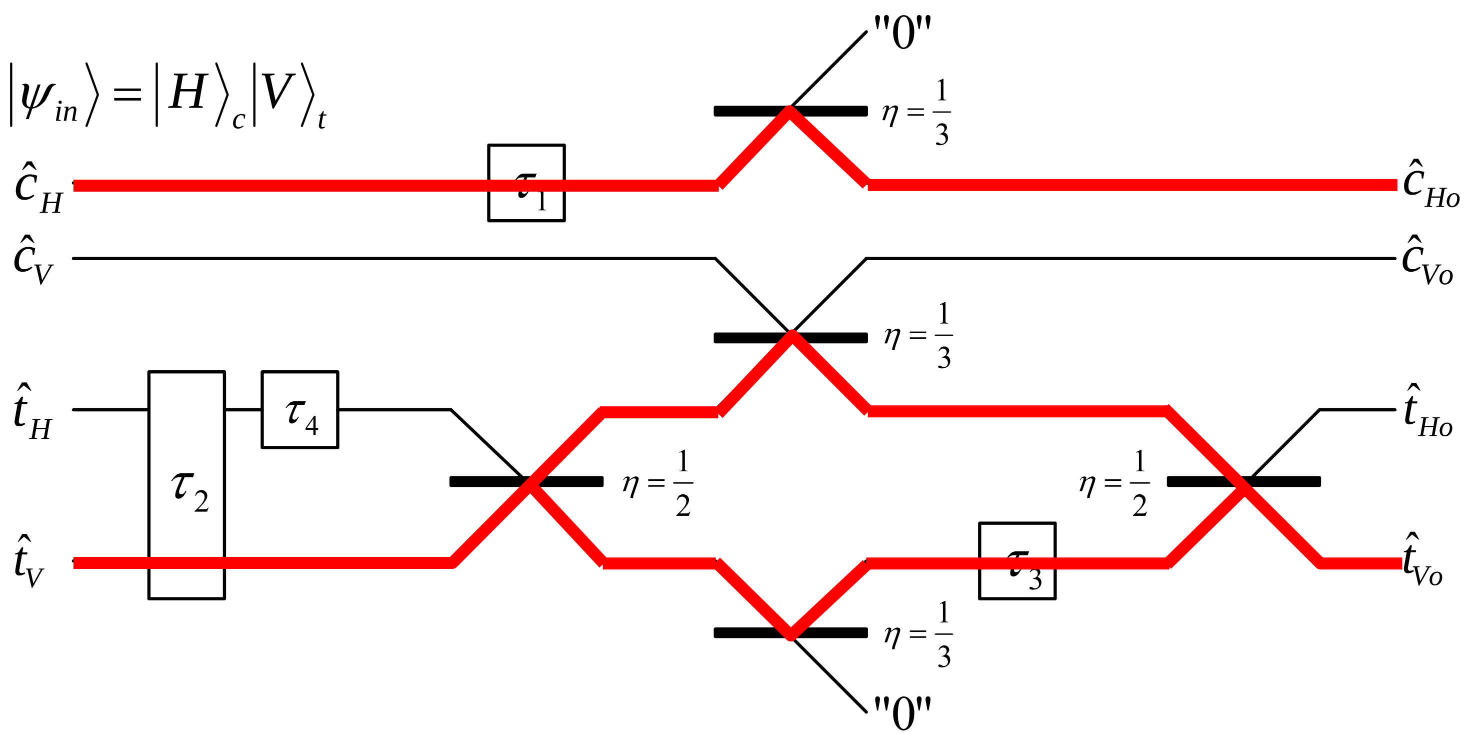


Peter’s question of the day

Can we infer the mode-matching parameters?

Non-intrusive determination of mode-matching parameters

- Assumptions:
  - Detectors are ideal → no filtering
  - Only one degree of freedom in mode-matching



Measure in

$|H\rangle_c |V\rangle_t$

basis

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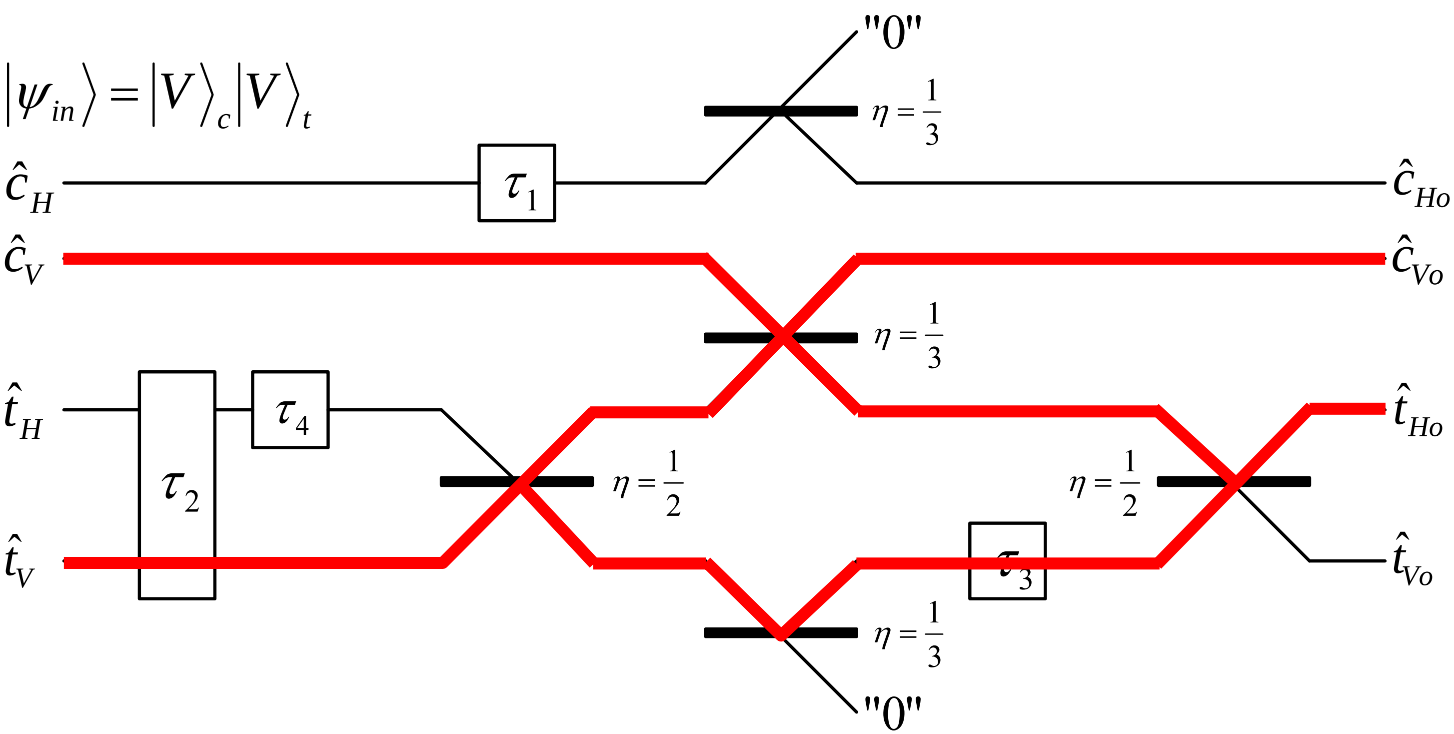

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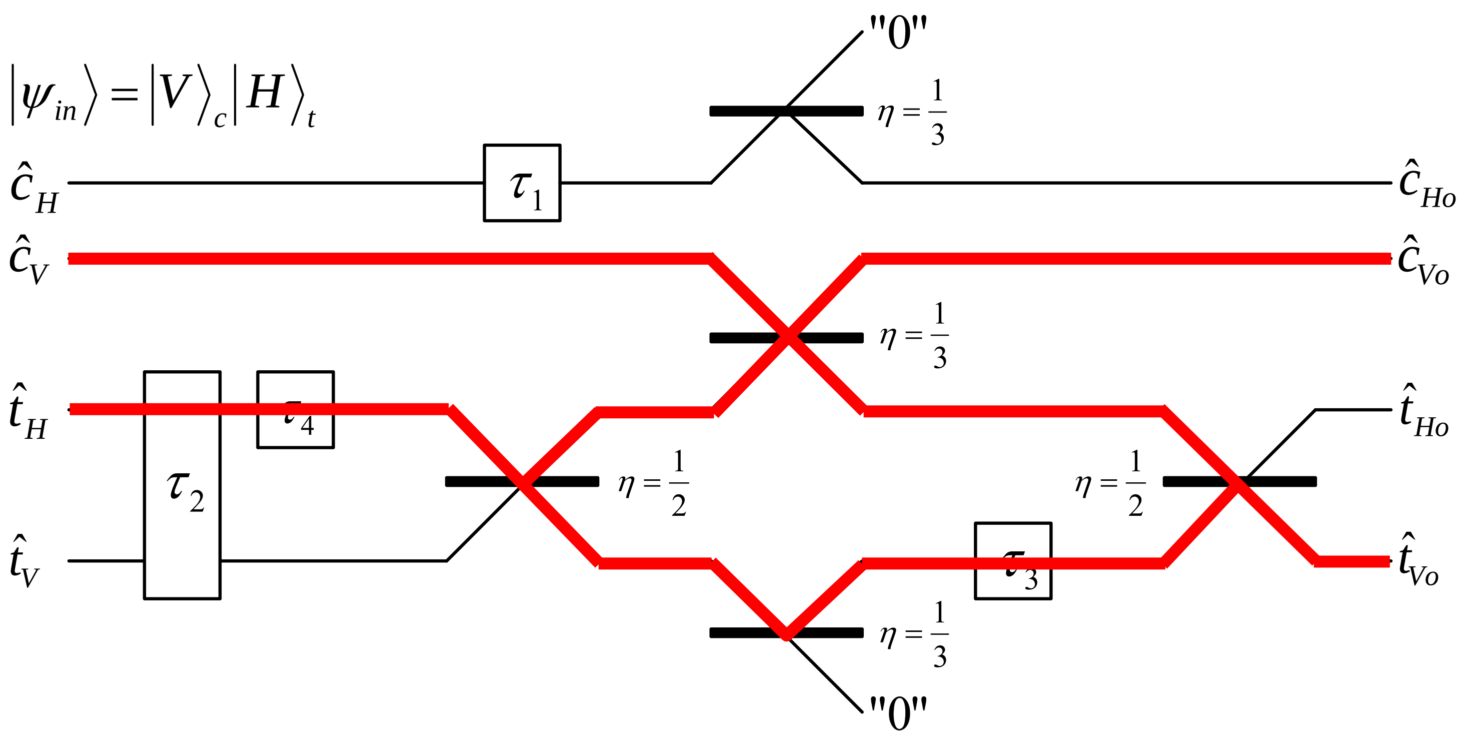

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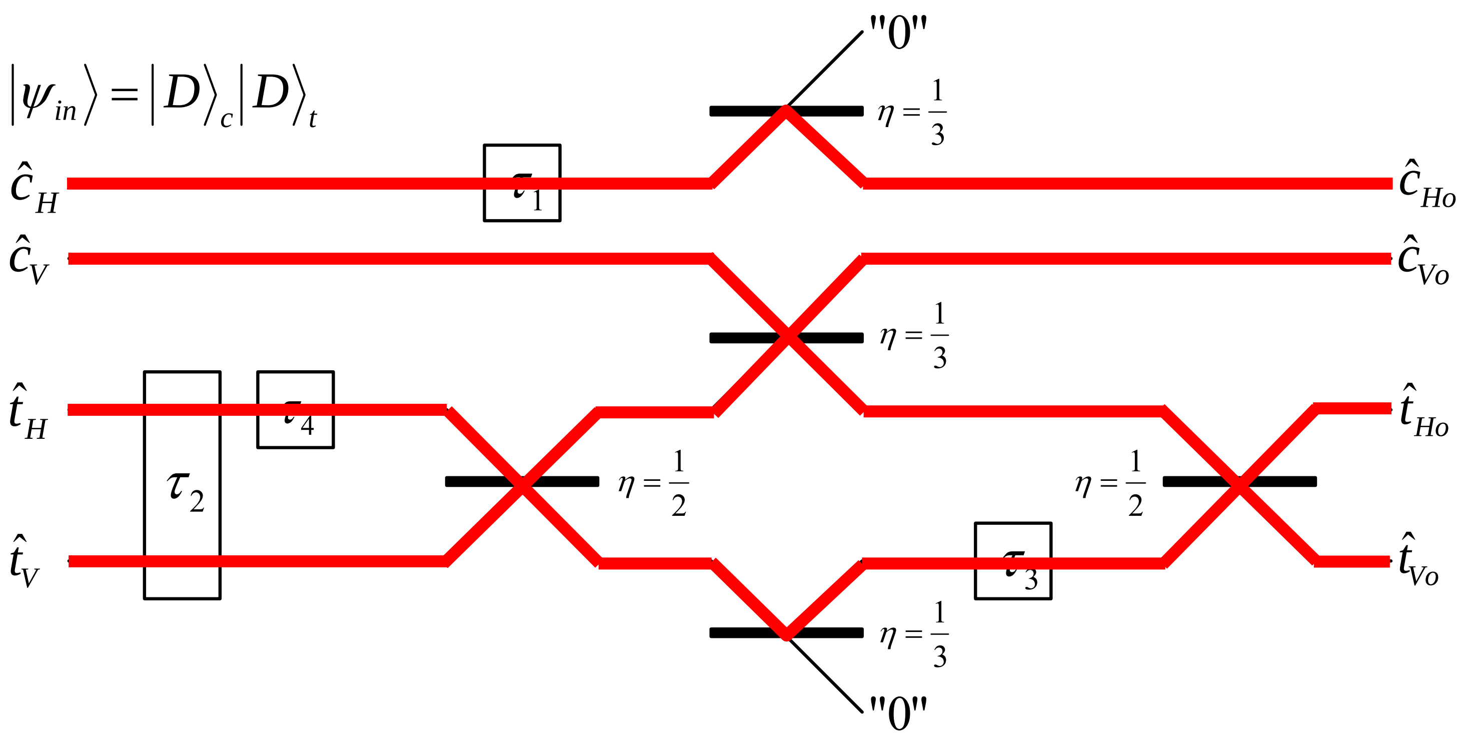

Measure in

$|D\rangle_c |D\rangle_t$

basis

Non-intrusive determination of mode-matching parameters

- In principle, 4 coincidence measurements can uniquely determine τ_1-τ_4 non-intrusively
- Estimates of τ_1-τ_4 not optimal!
- Problems
  - We assumed that there is only one degree of freedom in mode-matching
  - Our 4 measurements might be affected by multiple-mode matching effects

Non-intrusive determination of mode-matching parameters

- More general approach
  - Generate a matrix containing complete set of coincidence measurements and a complete set of input states

input states (←) ; coincidence basis (↓)

$$M_{expected} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \qquad M_{actual} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

- Perform optimization over τ_1-τ_4 to find least squares fit between $M_{expected}$ and M_{actual}
- Results in significantly more optimal estimates for τ_1-τ_4

Detector effects in coincidence-basis CNOT-gate

- Model detectors as before
- No mode-mismatching, but steering

$$|\psi_{in}\rangle = |V\rangle_c |H\rangle_t$$
$$\tau_{control} = 1, \tau_{target} = -1$$

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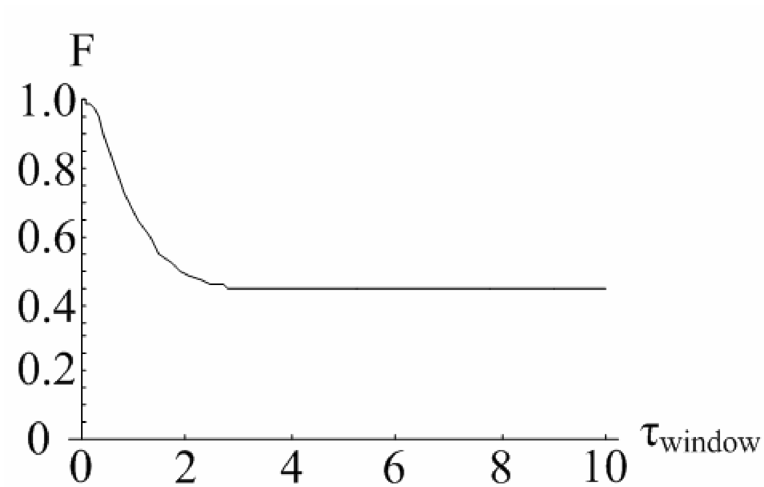

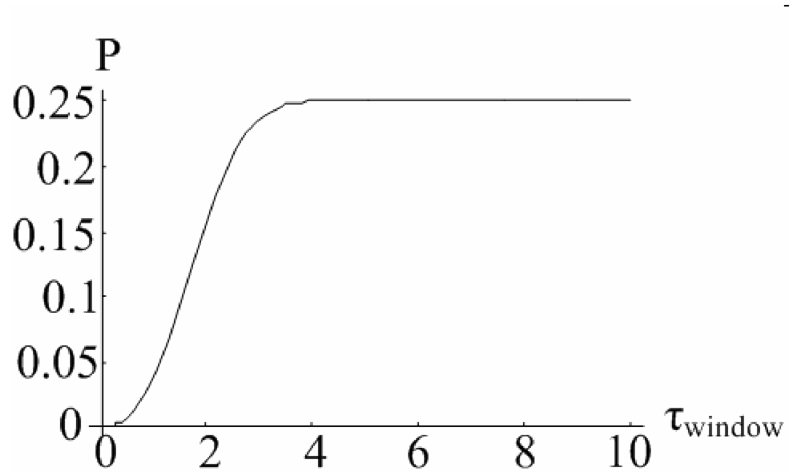

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$$\tau_{control} = 1, \tau_{target} = -1$$

Summary

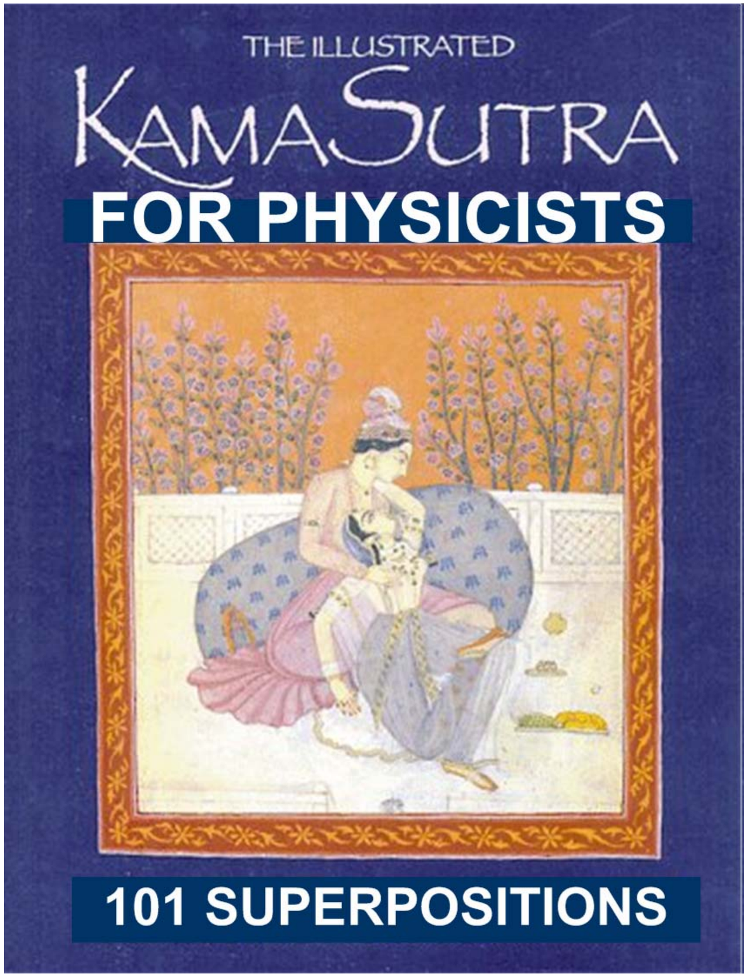
- Introduced general representation for photons
- Heralded CNOT-gate
  - Effects of distinguishable inputs
  - Detector effects
    - 'Broader' detectors are better
- Coincidence-basis CNOT-gate
  - Temporal-mode matching effects
  - Techniques for non-intrusive determination of mode-matching parameters
    - Leads to a more accurate model
  - Detector effects
    - 'Narrower' selective detectors are better

Final rant

- Techniques described apply in general to any source of distinguishability
  - Temporal
  - Frequency
  - Spatial
  - Polarization

Peter's lame joke of the day…

My idea for a book



Questions?