Mode-matching effects in linear optics quantum computing

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Introduction

It has recently been shown that efficient quantum computation can be implemented using only linear optics, single photon sources and single photon detectors, a scheme referred to as linear optics quantum computing (LOQC)[1]. Several in-principle experimental demonstrations of LOQC have recently been performed[2,3,4]. In the LOQC scheme qubits are encoded by the presence of a single photon in one of two orthogonal modes.

All LOQC implementations inherently require that the photons constituting the qubits be indistinguishable. A major problem facing all experimental LOQC implementations is *mode-matching*, whereby photon indistinguishability is compromised within a circuit and thus the circuit no longer operates ideally. Here we present a model of a controlled-NOT (CNOT) gate which explicitly allows for mode-matching. We show that this model has predictive power vastly superior to what present models allow. Additionally, the model can be used to non-intrusively estimate the mode-matching characteristics of an experimental gate. The techniques we describe are non-specific and could be applied to other LOQC circuits.

The ideal CNOT gate

Figure 1 illustrates the construction of the CNOT gate using beamsplitters with reflectivities η as indicated. The gate acts on two qubits, the control (c) and target (t) and ideally implements the logical transformation

$$\begin{aligned} |0\rangle_c|0\rangle_t &\to |0\rangle_c|0\rangle_t \\ |0\rangle_c|1\rangle_t &\to |0\rangle_c|1\rangle_t \\ |1\rangle_c|0\rangle_t &\to |1\rangle_c|1\rangle_t \\ |1\rangle_c|1\rangle_t &\to |1\rangle_c|0\rangle_t \end{aligned} \tag{1}$$

on the logical basis states, where $|0\rangle_L \equiv |0\rangle_H|1\rangle_V$ and $|1\rangle_L \equiv |1\rangle_H|0\rangle_V$.

The gate is non-deterministic and is post-selected upon detecting a total of exactly one photon between the control modes and one between the target modes[5].

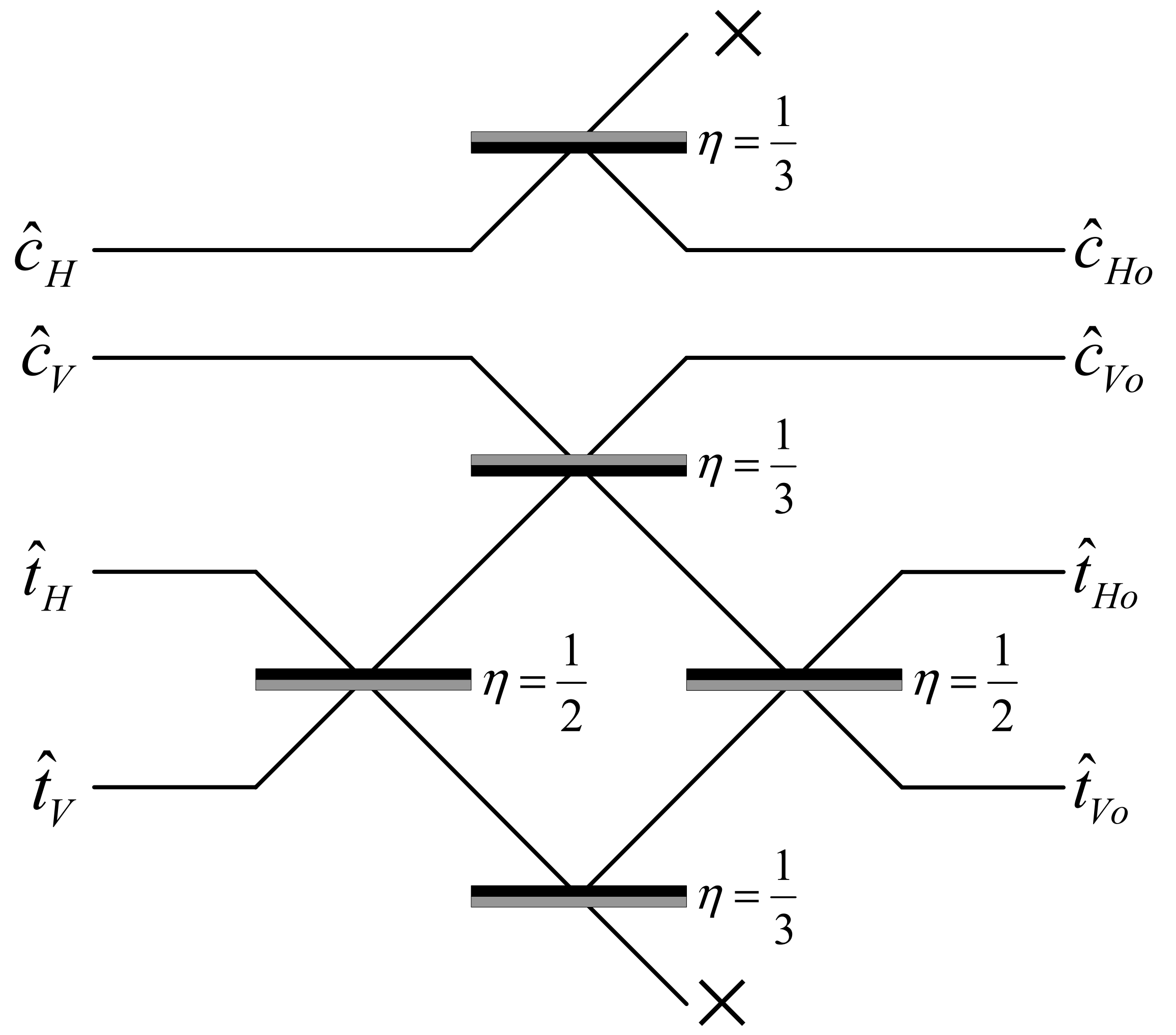

Figure 1. Layout of the CNOT gate using beamsplitters with reflectivities η. '×' are dump-modes.

Representing single photons

In order to observe mode-matching effects in our model we must first introduce a representation for photons which explicitly captures their spatio-temporal structure. We introduce the notation

$$|\psi\rangle_{photon} = \int \kappa(k)|1\rangle_k \, \mathrm{d}k \tag{2}$$

where k is some photon degree of freedom, $\kappa(k)$ is the normalised distribution function of the photon over that degree of freedom, and $|1\rangle_k$ is the single photon state at a particular location in the k degree of freedom. It can be shown that a single photon degree of freedom is sufficient to model arbitrary mode-matching effects.

CNOT gate with mode-matching

Photon distinguishability (*i.e.* mode-mismatch) can be introduced experimentally in many ways, including:

- Imperfect spatial overlap between photons
- Unsynchronised arrival time between photons
- Photons with different centre frequencies or bandwidths
- Photons with differing polarisations
- Or, anything else which gives away information about which photon is which

We model mode-matching by displacing the photon distribution functions at different locations in the circuit. Figure 2 illustrates this, where $\tau_1 \dots \tau_5$ are the displacement parameters.

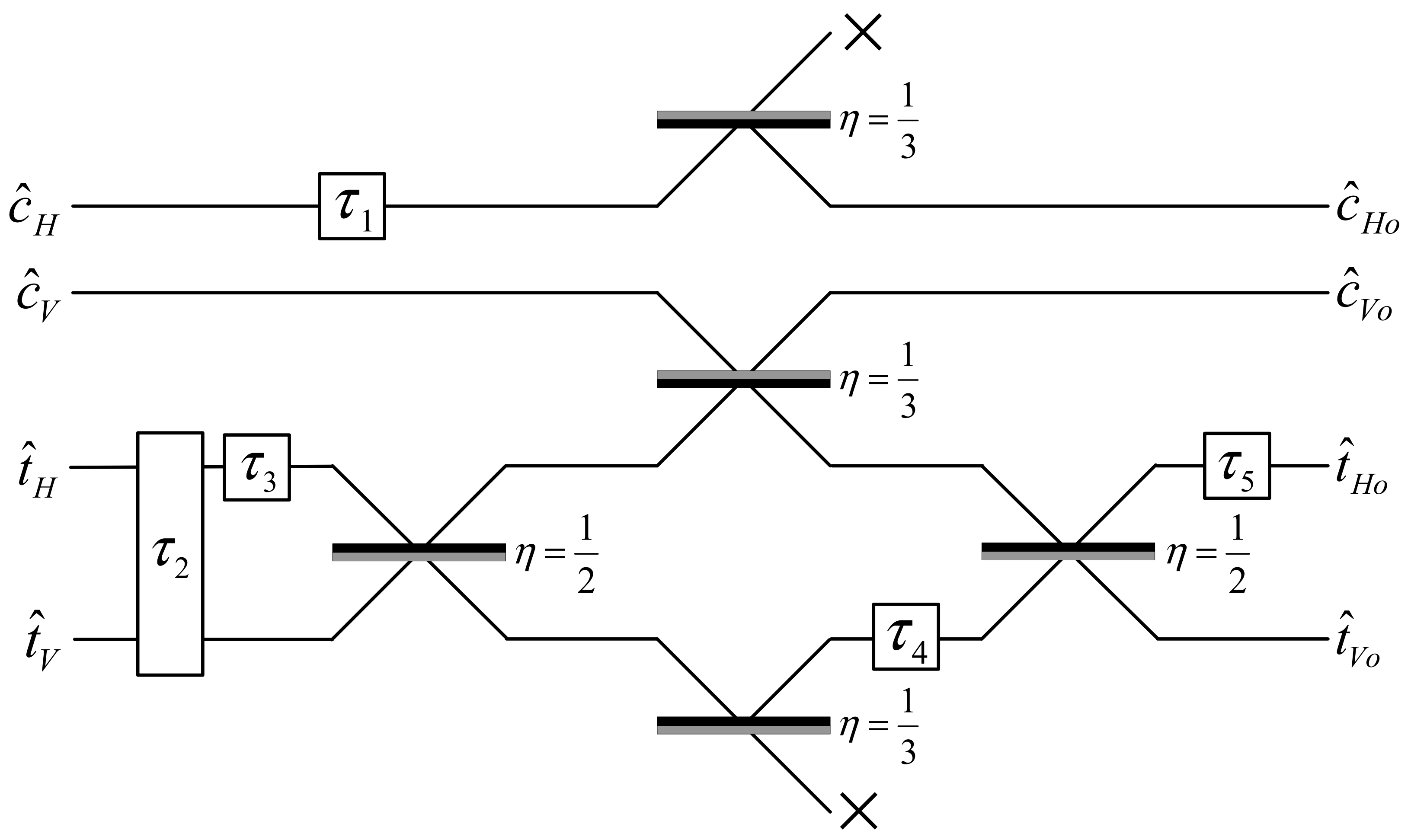

Figure 2. CNOT gate model with mode-matching, where $\tau_1 \dots \tau_5$ are the displacement (or mode-matching) parameters.

Estimating the mode-matching characteristics of an experimental gate

Using the gate model described it is possible to non-intrusively infer the mode-matching characteristics of an experimental gate. The procedure is as follows:

- From the gate model develop expressions for different measurement outcomes
- Perform the corresponding measurements on the experimental gate
- Optimise across $\tau_1 \dots \tau_5$ to find the best fit between the model and the experimental gate
- Substitute $\tau_1 \dots \tau_5$ back into the model to construct the optimised model

We perform this procedure twice: once optimising the τ-parameters globally, and once optimising them separately for each input state. The accuracy of the models are summarised in Table 1.

Gate model	$\mathscr{E}_{\max}$	$\mathscr{E}_{\mathrm{mean}}$
Ideal	15.25%	3.26%
Optimized (globally)	2.97%	1.32%
Optimized (decoupled inputs)	1.94%	0.67%

Table 1. Maximum ($\mathscr{E}_{\max}$) and average ($\mathscr{E}_{\mathrm{mean}}$) errors between the experimental gate[2] and the ideal (Figure 1) and optimised (Figure 2) gate models respectively.

The improvement present in the later optimisation is indicative that input state dependent (also known as *steering*) effects are playing a role in loss of gate fidelity, which may have been introduced during state preparation.

Conclusion

We introduced a model for an LOQC CNOT gate which explicitly allows for mode-matching effects. The model has predictive power superior to present ideal gate models (*i.e.* Figure 1) and additionally can be used to non-intrusively infer the mode-matching characteristics of an experimental gate (Table 1). The model has several differences to Quantum Process Tomography[6]:

- The parameters have a direct physical interpretation
- The model always predicts pure states
- The model is inherently physical (unlike QPT)

On the other hand, the model presented is limited to mode-matching errors, whereas QPT is a more general approach which makes no assumptions about the source of errors.

References

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