Quantum superpositions and correlations in coupled atomic-molecular BECs

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Quantum superpositions and correlations in coupled AM BECs.
Outline

- Coupled atom-molecular BECs – mechanisms
- Recent experiments at JILA
  - coherent oscillations in a BEC
  - quantum superposition of atoms and molecules
- Theory behind the superposition
  - predictions of 1998 revisited...
- Quantum correlated twin atom-laser beams
Why coupled AM BECs

- Can a molecular BEC form \textit{coherently} from an atomic BEC?
- New regimes of non-linear atom optics
- BOSE-ENHANCED chemistry or ‘\textit{superchemistry}’ at ultralow T
- New microscopic BEC physics to emerge

Quantum superpositions and correlations in coupled AM BECs.
**Route 1: Raman Photoassociation**

- Pairs of atoms collide, absorb a photon and emit one, giving a ground molecular state.

- Phase locking the two lasers ensures that the conversion is coherent (A BEC $\rightarrow$ M BEC).

- Drawback: spontaneous emission losses; weak free-bound couplings.

\[ 
\begin{align*}
E_2 & \quad \omega_2, \Omega_2 \\
E_3 & \quad \omega_1, \Omega_1 \\
E_1 & \quad 2E_1
\end{align*}
\]

\[ \delta, 2\Delta_1 \]

- Atomic BEC

- Molecular BEC

[Wynar et al., Science 287,1016 (2000)]
Route 2: Feshbach Resonance

- Tunable with an external magnetic field
- Bound molecular levels become resonant with the energy of free atoms
- Drawback: large losses due to inelastic collisions
- Advantages: stronger couplings; tunable atom-atom interactions

Quantum superpositions and correlations in coupled AM BECs.
JILA experiment

- \( {\text{Rb}}^{85} \) BEC with \( a > 0 \) near Feshbach resonance:
  
  \[
a \rightarrow a(B) = a_{bg} \left( 1 - \frac{\Delta B}{B - B_0} \right)
  \]

- For \( na^3 \sim 1 \) (rather than \( \ll 1 \)), the customary mean-field theory description breaks down

- New microscopic BEC physics to be expected!
Experimental procedure

- Prepare an atomic BEC at $B \simeq 166$ G, $a > 0$
- Apply a sequence of two fast magnetic-field pulses
- Measure the number of atoms remaining as a function of $t_{\text{evolve}}$, for different $B_{\text{evolve}}$

![Diagram showing the experimental procedure with pulse times and $B_{\text{evolve}}$]
Quantum superpositions and correlations in coupled AM BECs.
Atomic Ramsey interferometry

- Consider two-level atom ($|g\rangle$, $|e\rangle$); Apply two successive $\pi/2$ pulses, $R_1$ and $R_2$, at $\omega \simeq \omega_{ge}$.

- The first $R_1$ pulse transforms:

  $$|g\rangle \rightarrow (|g\rangle + |e\rangle) / \sqrt{2}$$

  $$|e\rangle \rightarrow (-|g\rangle + |e\rangle) / \sqrt{2}$$

- After a time delay $t_{\text{evolve}}$, $R_2$ transforms:

  $$|g\rangle \rightarrow (|g\rangle + |e\rangle e^{i\theta}) / \sqrt{2}$$

  $$|e\rangle \rightarrow (-|g\rangle e^{-i\theta} + |e\rangle) / \sqrt{2}$$

  $$\theta = (\omega - \omega_{ge}) t_{\text{evolve}}$$ - accumulated phase difference

- Probability of $|g\rangle \rightarrow |g\rangle$ or $|g\rangle \rightarrow |e\rangle$:

  $$P_{g\rightarrow g} = (1 + \cos \theta) / 2$$

  $$P_{g\rightarrow e} = 1 - P_{g\rightarrow g} = (1 - \cos \theta) / 2$$
Effective quantum field theory

\[ H_0 = \sum_{i=1,2} \int dx \left[ \frac{\hbar^2}{2m_i} |\nabla \hat{\Psi}_i|^2 + V_i(x) \hat{\Psi}_i^\dagger \hat{\Psi}_i \right] \]

\[ H_{\text{int}} = \frac{\hbar \chi}{2} \int dx \left[ \hat{\Psi}_2 \hat{\Psi}_1^\dagger \hat{\Psi}_1^\dagger + H.c. \right] \]

\[ H_{\text{self}} = \sum_{ij} \frac{\hbar U_{ij}}{2} \int dx \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i \]

- \( \hat{\Psi}_{1/2}(t,x) \) – atomic/molecular field operators
- \( U_{11} \propto a \) – atom-atom \( S \)-wave scattering
  – comes from \( \tilde{U}_{11}(x-y) \rightarrow U_{11} \delta(x-y) \), together with a UV momentum cutoff \( k_{\text{max}} \)
- \( \chi \) – atom-molecule coupling (\( A + A \rightleftharpoons A_2 \))
- \( V_i \) – trap potentials, including internal energies \( E_i \)
... apparatus 1 ...
Quantum superpositions and correlations in coupled AM BECs.
High density regime: 3D matter wave solitons

- At high densities (mean-field theory applies) wave-like interactions between the entire atomic and molecular BECs are favored
- Stable 3D matter-wave solitons can form in free space!

[PR 81, 3055 (1998)]
‘Superchemistry’ oscillations

- Start with a pure atomic BEC; switch on the coupling
- Giant coherent oscillations are observed between the atomic and molecular BECs
- Enhanced chemical reaction rates due to bosonic stimulation at $T \to 0$!

[PRL 84, 5029 (2000)]
Low-density regime: Coherent quantum superposition

- EXACT quantum ground state (for $N = 2$):

$$|\Psi^{(2)}\rangle = \left[\hat{b}^\dagger(0) + \int_{|k|=0}^{k_{\text{max}}} dk G(k) \hat{a}^\dagger(k) \hat{a}^\dagger(-k)\right]|0\rangle$$

- the eigenstate is a coherent superposition of a molecule with a pair of atoms: “dressed” molecule

- Energy eigenvalue

$$E = E_m - \frac{\hbar \chi^2}{2} \left[U_{11} + \frac{2\pi^2 \hbar R/m}{Rk_{\text{max}} - \tan^{-1}(Rk_{\text{max}})}\right]^{-1}$$

$$= -\frac{\hbar^2}{mR^2}.$$  

$R$ - correlation radius; $E_b = -E$ - binding energy.
Approximate quantum many-body solutions

Quantum many-body ($N > 2$) ground state is well approximated by:

$$\Psi^{(N)} = \left[ b^\dagger(0) + \int_{|k|=0}^{k_{\text{max}}} dk G(k) \hat{a}^\dagger(k) \hat{a}^\dagger(-k) \right]^{N/2} |0\rangle$$

- Corresponds to a gas of $N/2$ independent “dressed” molecules, with total energy

$$E^{(N)} = \frac{N}{2} E.$$  

- The two-particle solution $|\Psi^{(2)}\rangle$ and the corresponding energy $E$ are keys to understanding the JILA experiment!
Binding energy $E_b$ vs $B$?

- $E_b(B)/\hbar$ should give the frequency of oscillations observed (analogous to Ramsey fringes), as interference between atoms and “dressed” molecules.

$$E_b = -E_m + \frac{\hbar \chi^2}{2} \left[ U_{11} + \frac{2\pi^2 \hbar R/m}{R k_{\text{max}} - \tan^{-1}(R k_{\text{max}})} \right]^{-1}$$

$$= -\frac{\hbar^2}{m R^2}.$$  

- can be used for large $R$, with $R^{-1} \ll k_{\text{max}} \lesssim |a_{bg}|^{-1}$

- In JILA experiment, however, $R \sim |a_{bg}|$.

- Must carry out renormalization, to make the theory cut-off independent.
Renormalization

- Prevents UV divergencies as $k_{\text{max}} \to \infty$

- Relates ‘bare’ coupling constants in $H$ to the observed values

\[
\chi = \Gamma \chi_0, \quad U_{11} = \Gamma U_0, \quad E_m = E_0 + \alpha \Gamma \hbar^2 \chi_0^2
\]

where

\[
\Gamma = (1 - \alpha \hbar U_0)^{-1}, \quad \alpha = mk_{\text{max}}/(2\pi^2 \hbar^2), \quad U_0 = 4\pi \hbar a_{bg}/m, \quad E_0 = \Delta \mu (B - B_0)
\]

[Kokkelmans & Holland, cond-mat/0204504]
$E_b$ vs $B$ – final result!

$$B = B_0 + \frac{1}{|\Delta \mu|} \left( E_b + \frac{C_1 \sqrt{E_b}}{1 + C_2 \sqrt{E_b}} \right)$$

**THEORY →**

**EXP. →**

Quantum superpositions and correlations in coupled AM BECs.
more on quantum correlations

Twin atom-laser beams via dissociation of a molecular BEC

- produce quantum correlated atomic beams with entangled atom pairs
- matter-wave analog of parametric down-conversion in nonlinear optics
  \[ \frac{\omega}{2} \xrightarrow{\chi^{(2)}} \frac{\omega}{2} \]
- strong particle number-difference squeezing is expected
- applications in precision (sub-shot noise) measurements; new regimes of EPR & Bell correlations with massive particles

Quantum superpositions and correlations in coupled AM BECs.
Photo-dissociation of a molecular BEC

- start with a BEC of molecular dimers
- **coherently** dissociate the molecules into atom pairs, using coherent Raman transitions

![Diagram showing photo-dissociation process]

- Energy conservation: $\hbar |\Delta| = \hbar^2 k^2 / 2m_1$

Momentum conservation: 

$$\pm k_0 = \sqrt{2m_1 |\Delta| / \hbar}$$

Quantum superpositions and correlations in coupled AM BECs.
Twin atomic beams (1D)

Quantum superpositions and correlations in coupled AM BECs.
Relative particle number squeezing

• Variance of fluctuations in particle number difference in two beams:

\[ V = \frac{\langle [\Delta (\hat{N}_- - \hat{N}_+)]^2 \rangle}{\langle (\hat{N}_- + \hat{N}_+) \rangle}, \]

\[ = 1 + \left[ \langle : (\hat{N}_+)^2 : \rangle - \langle \hat{N}_- \hat{N}_+ \rangle \right] / \langle \hat{N}_+ \rangle \]

• \( V \approx 0.07 < 1 \) (93% squeezing) in this example (losses allowed at 10%; dissociation time \( \sim 2 \) ms, \( \xi_0 \approx 30 \mu m \)
SUMMARY

• Coherent oscillations in BEC number in JILA Feshbach resonance experiment indicate presence of quantum superposition between atoms and molecules.

• Our model quantum field theory gives explicit analytic expression for the superposition state.

• The corresponding binding energy and the experimentally observed frequency of oscillations are in remarkable agreement!

• Dissociation of a molecular condensate can produce strongly correlated twin atomic beams, with almost perfect squeezing in relative particle number.