Motivation

- add parametric down-conversion of matter waves to the arsenal of nonlinear atom optics
- possess a resource similar to production of entangled photon pairs in optical down conversion

Quantum effects expected:

- strong particle number-difference squeezing in the twin atomic beam output
- EPR correlations and Bell inequalities with massive particles

Dissociation of a molecular BEC

- Start with a BEC of molecular dimers
- coherently photo-dissociate the molecules into atom pairs, using Raman transitions (reverse to Raman photo-association: [PRL 84, 5029(2000)])
- Energy conservation: \( n|\Delta| = \frac{s^2 k^2}{2m} \)
- Momentum conservation: \( \pm k_0 = \sqrt{2m}|\Delta|/h \)

Example of a molecular BEC

\( H_b = \sum \int dx \left( \frac{s^2}{2m_b} \nabla \hat{\Psi}_1^2 + \kappa \Delta \hat{\Psi}_1 \hat{\Psi}_1 + V_2 \hat{\Psi}_1^2 \hat{\Psi}_2 \right) \)

\( H_{im} = \frac{\hbar \kappa}{2} \int dx \left( \hat{\Psi}_2 \hat{\Psi}_1 \hat{\Psi}_1^* \hat{\Psi}_2 \right) + H.c. \)

\( H_{attr} = \sum \frac{\hbar \kappa}{2} \int dx \left( \hat{\Psi}_2 \hat{\Psi}_1 \hat{\Psi}_1^* \hat{\Psi}_2 \right) \)

- \( \hat{\Psi}_{1,2}(t, x) \) – atomic/molecular field operators
- \( \chi \) – atom-molecule conversion rate
- \( U_0 = 4\pi\kappa\alpha_0/m \rightarrow s\)-wave scattering
- \( 2\Delta = (2E_i - E_0)/h - (\omega_1 - \omega_2) \) – two-photon detuning (like phase mismatch in optics)
- include losses

Approximations

- neglect molecular field depletion: replace \( \hat{\Psi}_2 \) by \( c\)-number; assume a Thomas-Fermi profile for the molecular BEC density \( n_2 \): \( \chi(t) \hat{\Psi}_2(x) \rightarrow \chi(t) \sqrt{n_2(x)} = \chi(t, x) \Delta + U_0 \hat{\Psi}_2^2(x) \hat{\Psi}_1 \rightarrow \Delta + U_0 n_2(x) \approx \Delta(x) \)
- limit to short interaction times and small number of atoms produced, and neglect atom-atom s-wave scattering

Techniques

Solve numerically the positive-P stochastic differential equations with noise terms (all quantum effects are retained and treated exactly)

Relative number squeezing

- Quantum correlations:
  \( V = \langle |\Delta(N_- - (N_-)|^3 \rangle / \langle |N_- + (N_-)| \rangle \)
  \( = 1 + |\langle (N_0) \rangle^2 | - (N_-N_0)| / \langle N_0 \rangle \)

- normalized variance of fluctuations in the particle number difference (with \( \Delta X \equiv X - \langle X \rangle \), where \( N_0(t) = \int_0^{\infty} dx \hat{\Psi}_1^*(x) \hat{\Psi}_1(x) \)
  \( \rightarrow \) squeezing corresponds to: \( V < 1 \)
- Twin beams = spatial separation AND squeezing

Example of trapped beams

Two beams, NOT twins (\( V > 1 \))

Strong repulsive atom-molecule S-wave scattering: \( U_2 n_2(x) \) acts like a potential barrier.

Example of trapped beams

Strong attractive atom-molecule S-wave scattering: \( U_2 n_2(x) \) acts like a trapping potential

Summary

- scheme for generating strongly correlated twin atom-laser beams
- phase insensitive and robust against losses
- measurement of the particle number in one of the beams would produce a single beam with well-defined particle number

Characteristic parameter values

- \( \alpha_2 \approx \alpha_1 = 5.4 \) nm, \( \alpha_2 = -9.25 \) nm (\(^{87}\)Rb)
- \( \Delta = -2 \times 10^4 \) s\(^{-1}\)
- molecular BEC: 60 \( \mu \)m; aspect ratio: 100
- initial number of molecules: \( 1.5 \times 10^2 \)
- final number of atoms: 100; losses: 10%
- dissociation time: 27 ms
- squeezing: 93% (\( V = 0.07 \))