NOVEL SOLITONS:
PARAMPS AND ATOM LASERS

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NOVEL SOLITONS?

- Classical and quantum solitons
- Solitons in fibers
- 2D & 3D solitons in parametric amplifiers
- Optical switching with parametric solitons
- Gap parametric solitons
- Atom laser or BEC solitons: superchemistry
1. CLASSICAL AND QUANTUM SOLITONS

- **CLASSICAL VIEW:** A stable nonlinear waveform: \( \Phi(\tau) = \Phi(0)e^{i\omega \tau} \).

- Early example: Nonlinear wave in shallow water (KdV equation)

- Modern example: Short optical pulse in single-mode fiber

- **QUANTUM VIEW:** A field theory energy eigenstate: \( \hat{H}\Psi = E\Psi \).

- Early example: theoretical models in high-energy physics

- Modern example: Short optical pulse as a photonic bound state
UNSOLVED PROBLEMS IN SOLITONS

- Can we form classical solitons in more than one dimension?
- When can we form quantum solitons?
- Are there essential differences between them?
- Do we require integrable dynamical equations?
- Are there applications in communications or computing?
- Is there new physics, e.g. different quantum statistics?
2. SOLITONS IN OPTICAL FIBERS

Classical equations are approximately valid at large photon number:

\[
\frac{i \partial \phi}{\partial t} = -\frac{\hbar}{2m} \frac{\partial^2 \phi}{\partial z^2} + \chi \phi^{\dagger} \phi^2
\]

This is the (scaled) nonlinear Schroedinger (NLS) equation, where \( m = \hbar/\omega'' \) is the effective mass, and \( \chi \) is the nonlinearity. The soliton is classical, and is observed to propagate with solution \(|\phi| \simeq sech(\alpha z)|.\)
Quantum theory

There exists a set of bound states of this quantum field.

- Hamiltonian: \( \hat{H} = \frac{1}{2} \int [\hbar^2 |\partial \phi^\dagger / \partial z|^2 / m + \hbar \chi \phi^\dagger \phi^2] dz \)

- Di-photon eigenstate: \( |\psi \rangle \approx \int e^{-\lambda |z_1 - z_2|} \phi^\dagger (z_1) \phi^\dagger (z_2) d^2z |0 \rangle \)
How are these related to physical experiments?

- We often do experiments with $10^9$ photons or more.
- The bound states exist, but are not easily distinguished from each other.
- Simplest experiments observe classical soliton behaviour
- These are used in experimental communications systems (Mollenauer)
- Quantum effects first predicted by Carter & Drummond
- Quantum squeezing, collisions observed by: Shelby, IBM Labs; Friberg, NTT Labs; Haus, MIT; Bergman, Princeton; Leuchs & Sizman, Erlangen
3. NOVEL SOLITONS IN PARAMETRIC AMPLIFIERS

- Parametric solitons are theoretically stable in up to 3 dimensions
- Spatio-temporal solitons are particle-like ‘objects’, with two coupled fields
- Ultra-fast response time (fs) due to $\chi^{(2)}$ nonlinearity
- Low pulse energy (pJ); polarization scales as $E^2$, not $E^3$
Parametric equations

The scaled equations are given in the form:

\[ iu_\xi + \nabla^2_\perp u + u_{\tau\tau} - u + vu^* = 0, \]
\[ 2iv_\xi + \nabla^2_\perp v + \delta v_{\tau\tau} - \gamma v + \frac{1}{2}u^2 = 0, \]

- \( u \) and \( v \): the FH and SH envelopes
- \( \xi = z/z_0; \tau = t/t_0; \rho = r/r_0 \); \( \nabla_\perp = \partial^2/\partial \rho^2 + (d-1)/\rho \partial/\partial \rho \)
- \( \gamma \): phase mismatch; \( \delta = \omega''_2/\omega''_1 \): ratio of dispersion
Rigorous properties

The NLS equation is only stable in one dimension. BUT:

- For $\delta = 2$ and $\gamma \geq 4$, stable parametric soliton.
- Stable in one, two and three dimensions!
- Requires group velocity matching between harmonics
- Soliton cannot decay except through absorption
- Soliton collisions: phase-dependent, exchange energy
- Equations are *NOT* an integrable family
Variational method

- **Lagrangian density**

\[ \mathcal{L} = \frac{1}{2} \left[ (\nabla_{\rho} u)^2 + (\nabla_{\rho} v)^2 + u_{\tau}^2 + \delta v_{\tau}^2 + u^2 + \gamma v^2 - u^2 v \right] . \]

- **Gaussian ansatz**

\[
\begin{align*}
    u &= A \exp \left( -a \rho^2 - \alpha \tau^2 \right), \\
    v &= B \exp \left( -b \rho^2 - \beta \tau^2 \right).
\end{align*}
\]
Snapshots of 2D simultons
4. OPTICAL SWITCHING WITH PARAMETRIC SOLITONS

Collision of two different polarization pulses. If both arrive simultaneously, a soliton is generated. Otherwise, no soliton.

- Logical and gate with spatially separated inputs
- Time resolution: fs
- Energy requirement: pJ
- Phase insensitive; self-generated SH field
Optical switching

1 & 0 = 0

1 & 1 = 1
Numerical simulation:

PARAMETERS: \( d = 2, \gamma = 1, \rho = 1, \nu = \pm 0.1 \)
5. GAP PARAMETRIC SOLITONS

- Conventional $\chi^{(3)}$ medium – Low nonlinearity; low dispersion
- Paramp – High nonlinearity due to $\chi^{(2)}$ effects; low dispersion.
- Conventional grating – Low nonlinearity; high dispersion near band-gap.
- **GRATING PARAMP – HIGH NONLINEARITY & DISPERSION**
The coupled mode equation

\[
\begin{align*}
\left[\frac{i}{v_1} \frac{\partial}{\partial t} + \frac{i}{v_1} \frac{\partial}{\partial z} + \frac{1}{2k} \nabla_{\perp}^2\right] A_{1+} + \kappa_1 A_{1-} + \chi E A_{1+}^* A_{2+} &= 0 \\
\left[\frac{i}{v_2} \frac{\partial}{\partial t} - \frac{i}{v_2} \frac{\partial}{\partial z} + \frac{1}{2k} \nabla_{\perp}^2\right] A_{1-} + \kappa_1^* A_{1+} + \chi E A_{1-}^* A_{2-} &= 0 \\
\left[\frac{i}{v_2} \frac{\partial}{\partial t} + \frac{i}{v_2} \frac{\partial}{\partial z} + \delta k_2 + \frac{1}{4k} \nabla_{\perp}^2\right] A_{2+} + \kappa_2 A_{2-} + \chi E A_{2+}^2 &= 0 \\
\left[\frac{i}{v_2} \frac{\partial}{\partial t} - \frac{i}{v_2} \frac{\partial}{\partial z} + \delta k_2 + \frac{1}{4k} \nabla_{\perp}^2\right] A_{2-} + \kappa_2^* A_{2+} + \chi E A_{2-}^2 &= 0.
\end{align*}
\]

where: \(\chi_E = \omega_1^2 \chi^{(2)}/(k c^2)\), and \(\kappa_j = j k \Delta_{jj}/2\).
Envelope for the photon field

Define a photon field density for the bandgap modes:

\[ \Psi_j^{(s)}(x) = L^{-D/2} \sum_k a_{jk}^{(s)} e^{ik \cdot x}, \]

where: \( |\Psi_j^{(s)}(x)|^2 \) is the photon density for the \( j \)-th carrier in the \( s \)-th symmetry mode

- \( k = (0, 0, Q) \) in one dimension,
- \( k = (k_m, 0, Q) \) in two dimensions
- \( k = (k_m, Q) \) in three dimensions
EMA Hamiltonian

Using the effective mass approximation - valid for fields that propagate inside the band-gap - we have:

$$\frac{H}{\hbar} \approx \int \left[ \sum_{j,s} \left( \frac{s\hbar}{2m_j} |\partial_z \Psi_j^{(s)}|^2 + \frac{\hbar}{2m_j} |\nabla_{D \perp} \Psi_j^{(s)}|^2 + \omega_j^{(s)} |\Psi_j^{(s)}|^2 \right) \\
- \left( \frac{1}{2} \sum_{\vec{s}} \int \chi(\vec{s}) (\Psi_2^{(s_2)})^* \Psi_1^{(s_1)} \Psi_1^{(s_1')} + h.c. \right) \right] d^D x.$$
Dimensionless equations

The equation for two coupled modes in this Hamiltonian EMA approximation, is

\[
(i\partial_\tau + s_1 \partial_\zeta^2 + \nabla_{\rho_\perp}^2 - 1) V_1 + V_2 V_1^* = 0, \\
(i\sigma \partial_\tau + \tilde{s}_2 \partial_\zeta^2 + \nabla_{\rho_\perp}^2 - \gamma) V_2 + \frac{1}{2} V_1^2 = 0.
\]

THESE EQUATIONS ARE KNOWN TO HAVE COMPLETELY STABLE, ONE, TWO AND THREE DIMENSIONAL SOLITONS.
6. ATOM LASER OR BEC SOLITONS

Superchemistry BEC simultons

- Parametric coupling of two atoms to one molecule
- Theory predicts a non-diverging atom laser pulse?
‘Superchemistry’ coupling of atoms to molecules

- Raman photo-association converts two atoms into one molecule
Exact solutions: ‘dressed’ quantum molecular solitons

Stable for low densities (< $10^4$ atoms in trap).
Corresponds to $\frac{N}{2}$ molecules, each of which exist in a linear superposition with a pair of atoms (like a Cooper pair), so that the solution is:

$$|\psi_Q^N\rangle = \left[ \hat{b}^\dagger(0) + \int_0^{k_m} d^3k ~ g(k)\hat{a}^\dagger(k)\hat{a}^\dagger(-k) \right]^{N/2} |0\rangle .$$

Drummond, He, Kheruntsyan – Novel solitons: paramps and atom lasers
High density ‘classical’ molecular solitons

Corresponds to two coherently coupled Bose condensates - all chemical processes are enhanced by the Bose occupation numbers.

\[ |\psi_C^N\rangle = \exp \left\{ \int d^3x \left[ \phi(x)\hat{\Phi}^\dagger(x) + \psi(x)\hat{\Psi}^\dagger(x) \right] \right\} |0\rangle. \]

Stable for high densities (> 10^4 atoms in trap).

NEW TYPE OF COHERENT, NONLINEAR CHEMICAL KINETICS; BOSE-ENHANCED CHEMICAL REACTION RATES BECOME VERY LARGE NEAR ZERO TEMPERATURE.
Toroidal solitons

Toroidal solitons can form in a toroidal BEC magnetic trap, which has a ring-shape. These are the periodic solutions to the NLS equation. These solitons have a number of unusual properties:

- Vortex-type (integer winding number) solutions exist
- Other solutions have a phase-singularity, with winding number 1/2
- These ‘kink’ solutions have an approximate $\tanh(k\theta)$ behaviour
- They may be used to mode-lock an atomic ring laser.
SUMMARY

The field of solitons is very rich. Solitons can occur in non-integrable field theories, and in one, two, or three dimensions. Both quantum-like and classical-like behaviour is observable. As well as fundamental entities in nonlinear equations and quantum field theory, there are possible applications:

- Optical communications
- High-speed optical logic
- Stabilizing the output of an atom laser
- Modelocking an atom laser